A Consideration of Phase in Loudspeaker Design

A study of sources of phase and their influence on the on-axis and off-axis frequency response in the crossover region of a two-way loudspeaker

Charlie Laub, January 20, 2011

In order to obtain a balanced soundfield from a loudspeaker, the designer attempts to control the on-axis and off-axis frequency response. Typically the goal is to obtain smooth, flat frequency response on-axis, but less attention is paid to off-axis frequency response. Apart from the "direct" sound that travels in a straight line from the loudspeakers to the listener's ears, in a typical home environment the listener also is presented with a soundfield that is the sum of all of the off-axis sound radiated by the speaker and reflected off of the room environment. This indirect sound field can be as high as -3dB below the direct sound. As the listening area becomes large and the room boundaries move farther and farther away from the listener, the contribution from the off-axis sound diminishes, so the effect can be room-size dependent. However, in normal home listening spaces, the off-axis sound contribution is important and should not be neglected, since the loudspeaker's off-axis frequency response contributes to the overall spectral balance that the listener perceives.

Many factors influence the off-axis frequency response. Not all will be considered here. However one important area where deviations in the off-axis frequency response are common is the "crossover region", that is some range of frequencies above and below the crossover point where both drivers are operating with enough output to constructively and destructively interfere with each other, depending on the relative phase. By considering some of the common factors that influence the phase of each driver within the crossover region, and by studying how to control these factors, one can obtain useful information on how to minimize off-axis frequency response anomalies, and obtain a more balanced sound field.

A simple model for a driver's phase is a second order high-pass filter. For a driver in a closed box, or a tweeter, this is a good approximation to the phase response. The phase and amplitude for a range of 2nd order high-pass filters is shown in Figure 1. As the Q (quality factor) of the filter increases, the band of frequencies over which the phase changes from 180 degrees to zero becomes narrower and a peak emerges in the frequency response. Note that for low Q values such as Q=0.5, the phase change occurs over a very wide range of frequencies. The impact of this on the off-axis behavior will be demonstrated.

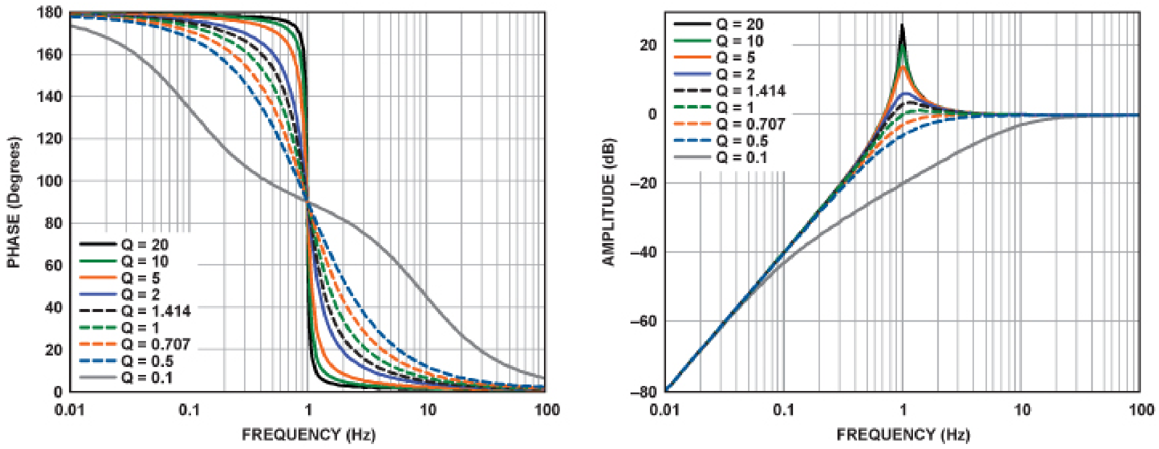


Figure 1: Second order high-pass filter phase (left) and amplitude (right).

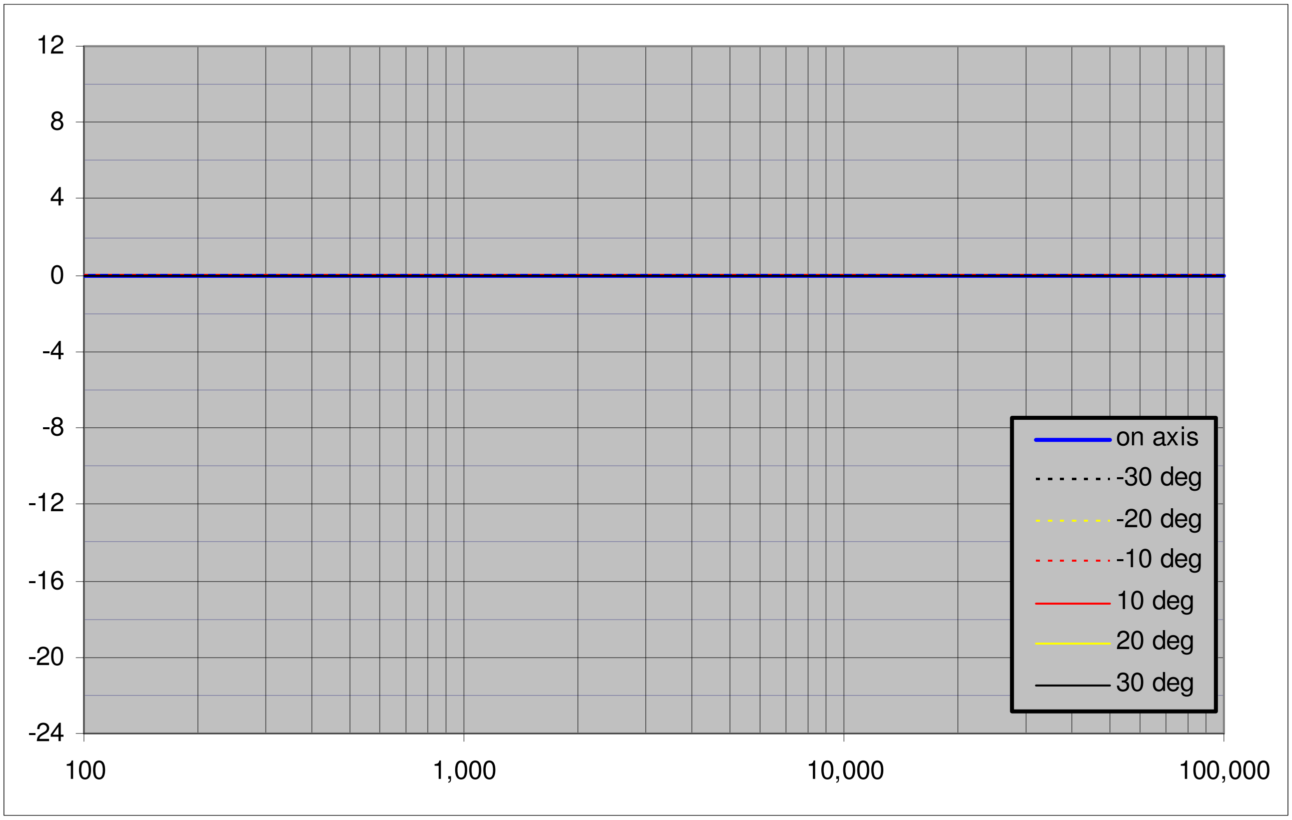
There are several sources of phase change in the crossover region:
(1) The separation of the drivers in the vertical direction (also known as "y-offset")
(2) The separation of the driver in the horizontal direction (also known as "z-offset")
(3) The driver's own phase response

INFLUENCE OF DRIVER SEPARATION (Y-OFFSET)

First, we will consider a simplified approximation of a two-way speaker: each driver is considered to be a point source. The acoustic center of both drivers will be assumed to be in the plane of the baffle, e.g. there is zero “z-offset”. The driver is modeled without considering phase, e.g. has constant phase and amplitude.

Let’s look at the influence of the separation of the drivers in the vertical plane (the “y-offset”) on the off-axis response, as a function of the number of wavelengths the drivers are separated at the crossover frequency. The crossover point, chosen to be 1000 Hz, is not a factor because the separation is given in terms of the number of wavelengths, so the result can be scaled to any crossover point. A second order Linkwitz-Riley (LR2) crossover will be used.

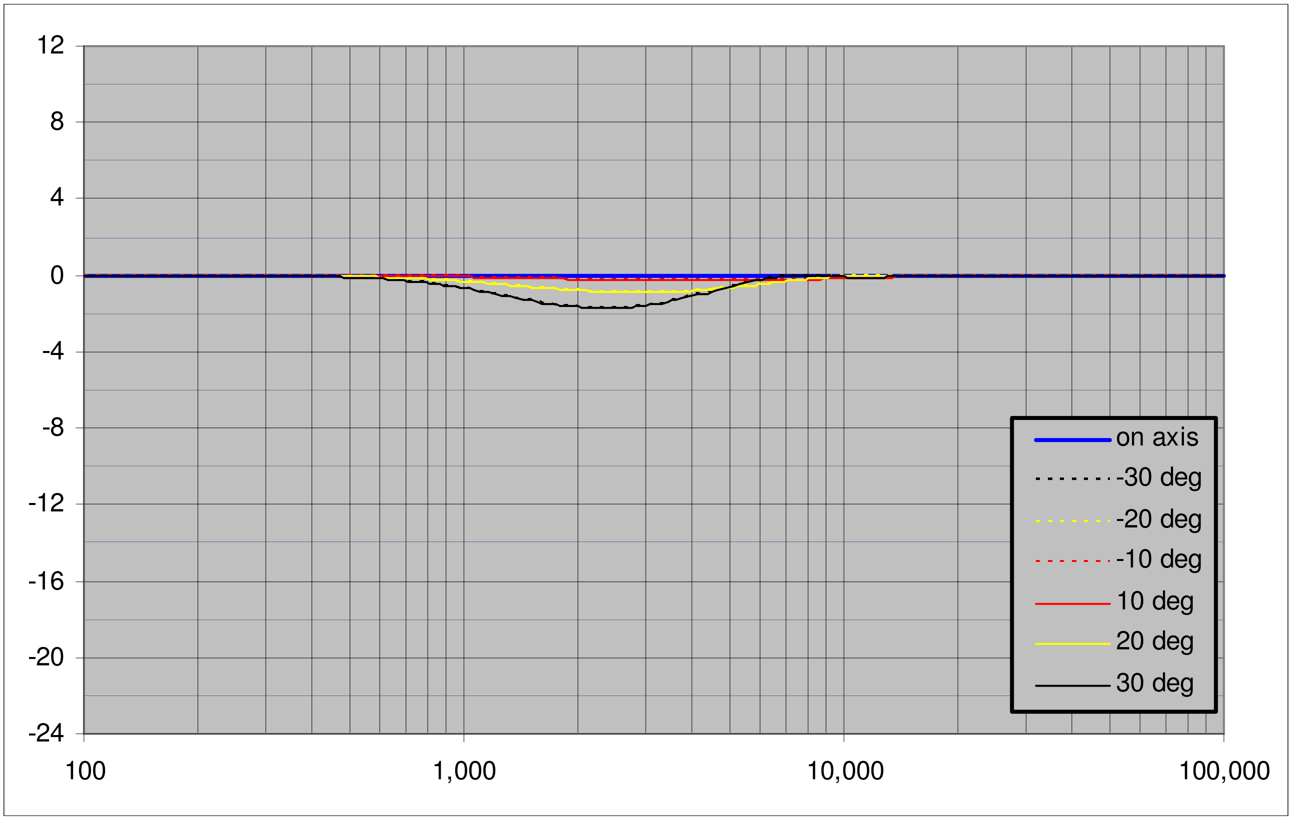
The first plot is the on-axis and off-axis responses when there is zero driver separation:



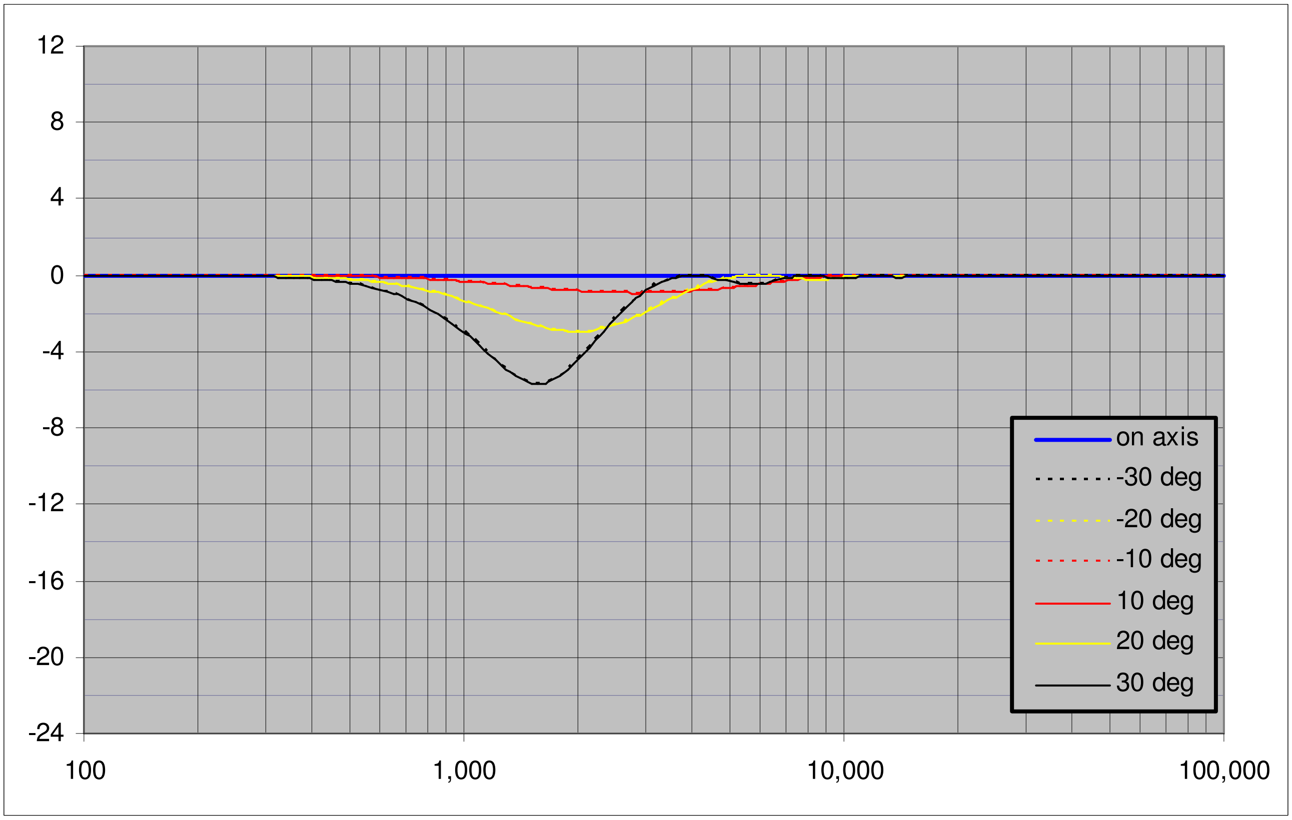
Not very interesting! Both the on-axis and off-axis responses are perfectly flat. This is because driver separation controls the resulting interference pattern. Because separation is zero, the pathlength from each driver to the listening point is the same and the drivers are completely in phase. No interference occurs.

When we start to move the drivers apart, the typical “lobing” pattern of the LR2 crossover emerges:

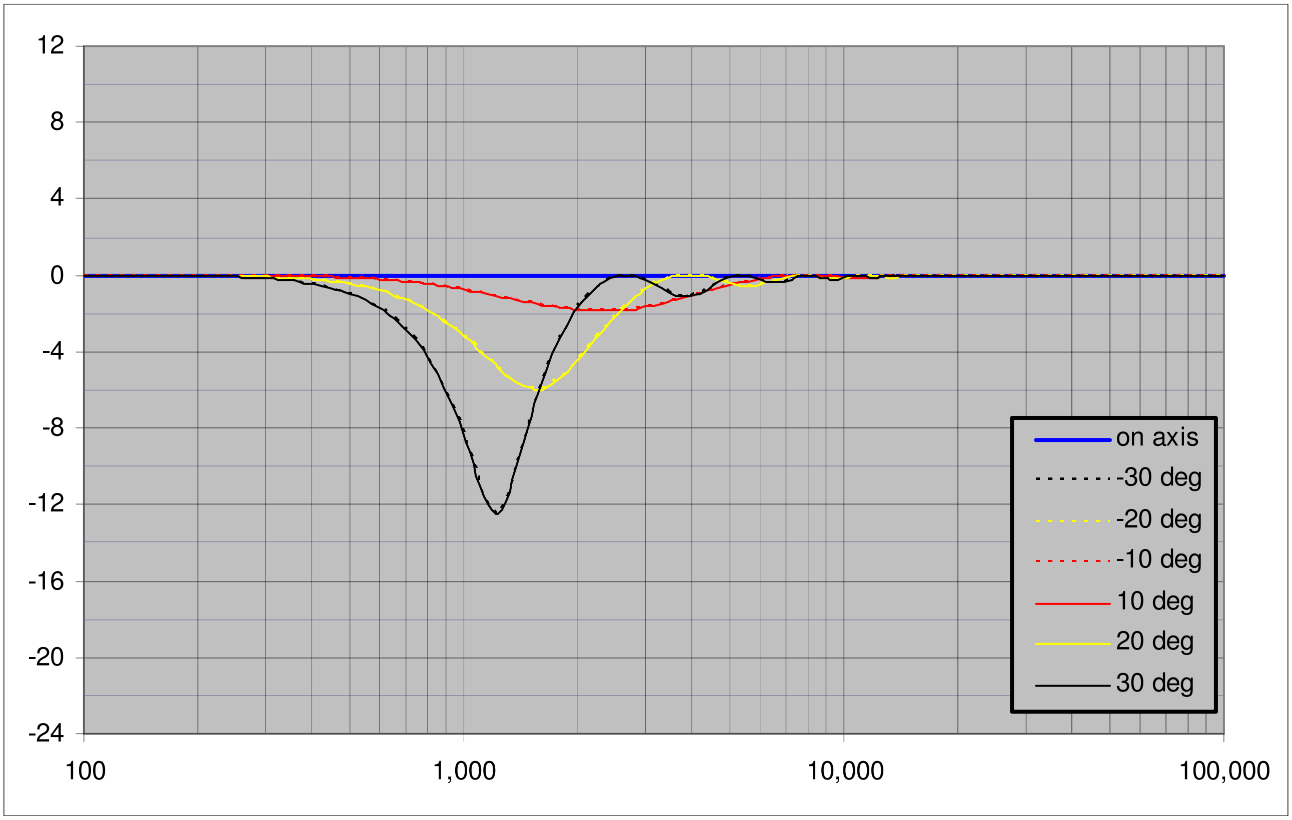
0.25 wavelengths y-offset:



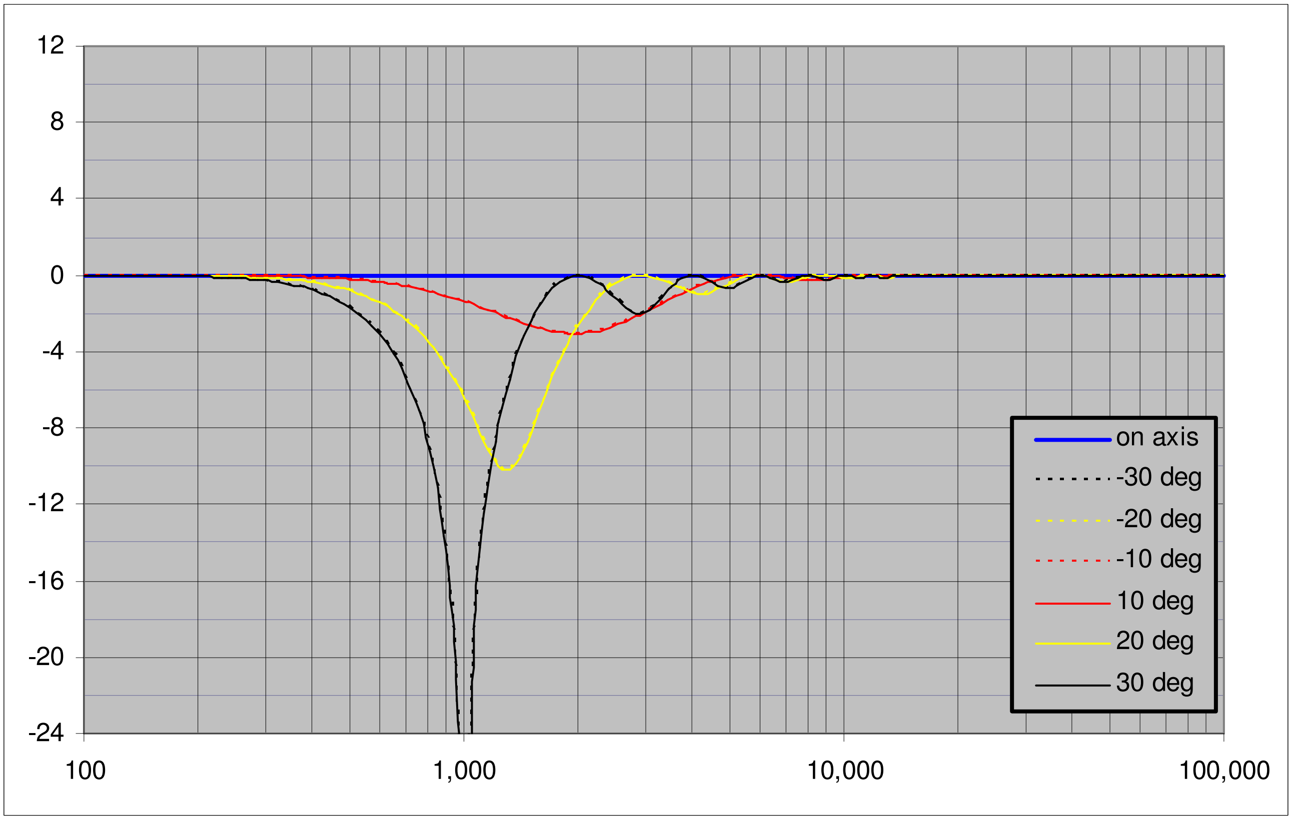
0.50 wavelengths y-offset:



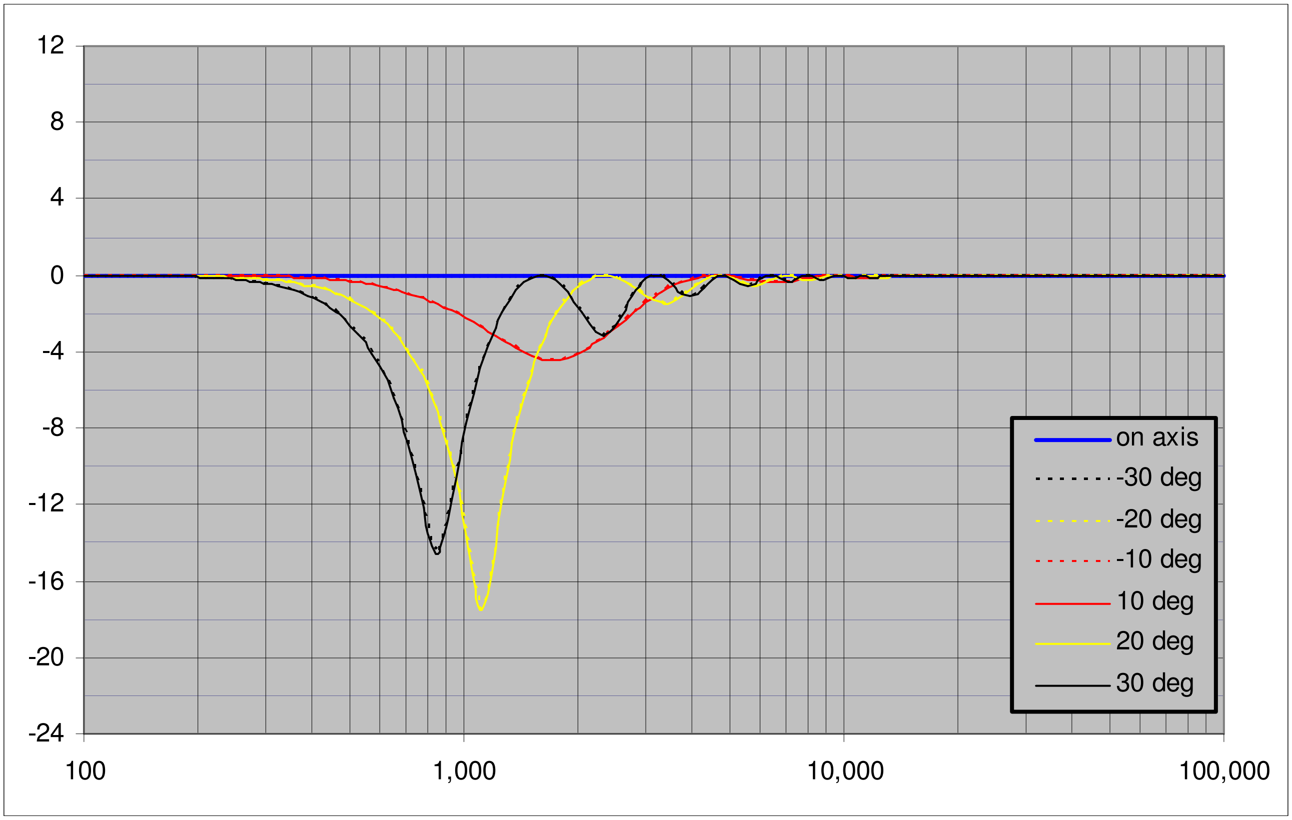
0.75 wavelengths y-offset:



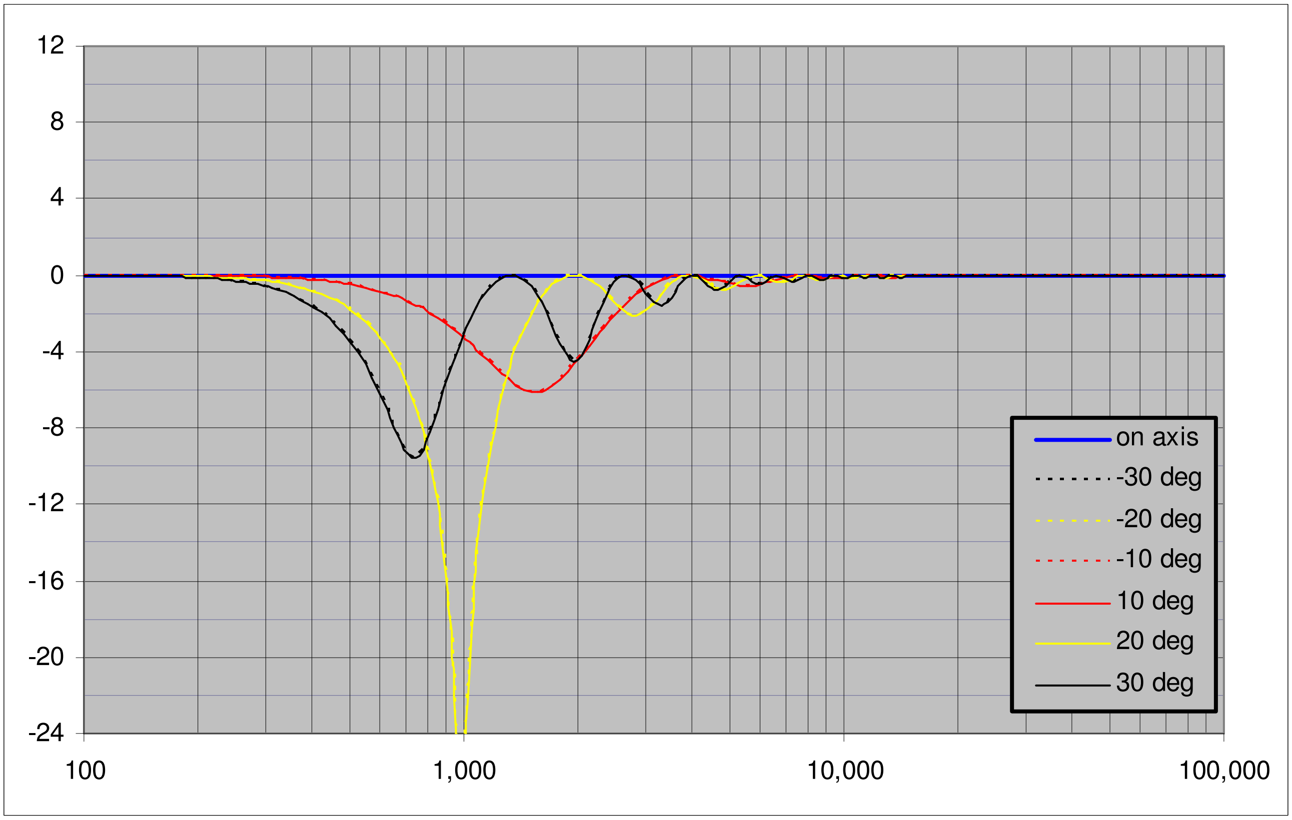
1.0 wavelengths y-offset:



1.25 wavelengths y-offset:



1.5 wavelengths y-offset:



As the y-offset driver spacing increases, the off-axis behavior has increasingly deep nulls at shallower angles. Note that the above axis (solid lines) and below axis (dashed lines) overlap for the same deviation off axis. This is a characteristic of Linkwitz-Riley type crossovers, which have symmetric off-axis frequency response.

In general, it can be observed that when the driver spacing is greater than approximately 0.5 wavelengths of the crossover frequency, the depth of the nulls within the 30 degree to -30 degree window become severe and multiple nulls may develop.

The following two tables list distances in meters (upper table) and inches (lower table) as a function of different frequencies and wavelength fractions:

$\lambda =$	0.25	0.5	1.0	1.5
500 Hz	0.1716	0.3432	0.6864	1.0296
1.0k Hz	0.0858	0.1716	0.3432	0.5148
1.5k Hz	0.0572	0.1144	0.2288	0.3432
2.0k Hz	0.0429	0.0858	0.1716	0.2574
3.0k Hz	0.0286	0.0572	0.1144	0.1716

$\lambda =$	0.25	0.5	1.0	1.5
500 Hz	6.8	13.5	27.0	40.5
1.0k Hz	3.4	6.8	13.5	20.3
1.5k Hz	2.3	4.5	9.0	13.5
2.0k Hz	1.7	3.4	6.8	10.1
3.0k Hz	1.1	2.3	4.5	6.8

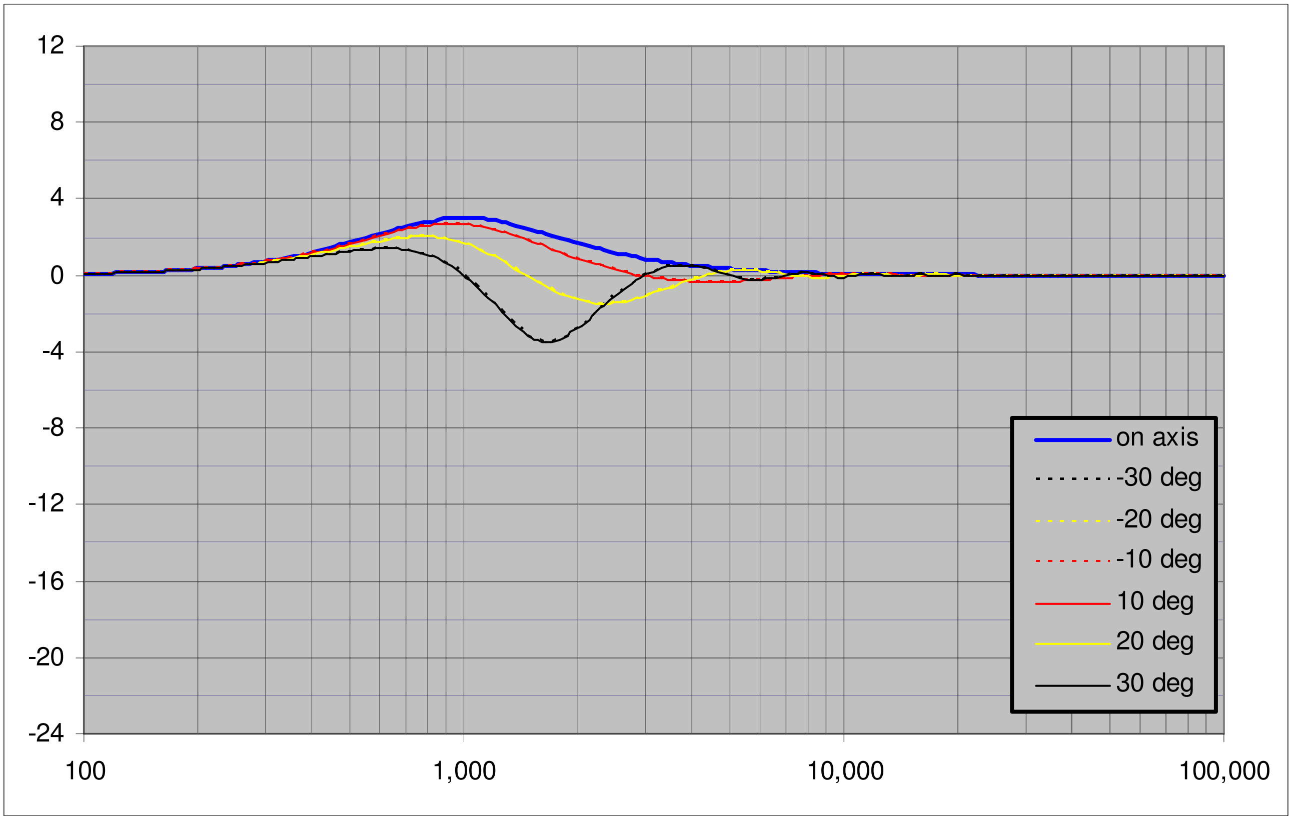
What can be observed from the tables above, is that it is virtually impossible for the distance between tweeter and mid/woofer to remain within 0.5 wavelengths at the crossover point unless the crossover point is low, e.g. less than 2k Hz. Even at 2k Hz, 0.5 wavelengths corresponds to only 3.4 inches and if a 1” dome tweeter with no flange was used the mid/woofer outer diameter would need to be less than or equal to about 5.5”. If a tweeter with the standard 4” flange were used, the midwoofer outer diameter is restricted to 3 inches or less! With careful selection of driver, these conditions can be met, however with increasing frequency the situation becomes hopeless. As a result, lower crossover point allow more flexibility in choosing a mid/woofer and locating it on the baffle, however the tweeter must then be capable of a very low crossover point (e.g. 1.5 k Hz).

Because the LR type crossovers have symmetric nulls and no peaks, the power response will also have a dip around the crossover frequency. For a good review of this, see the following web page:

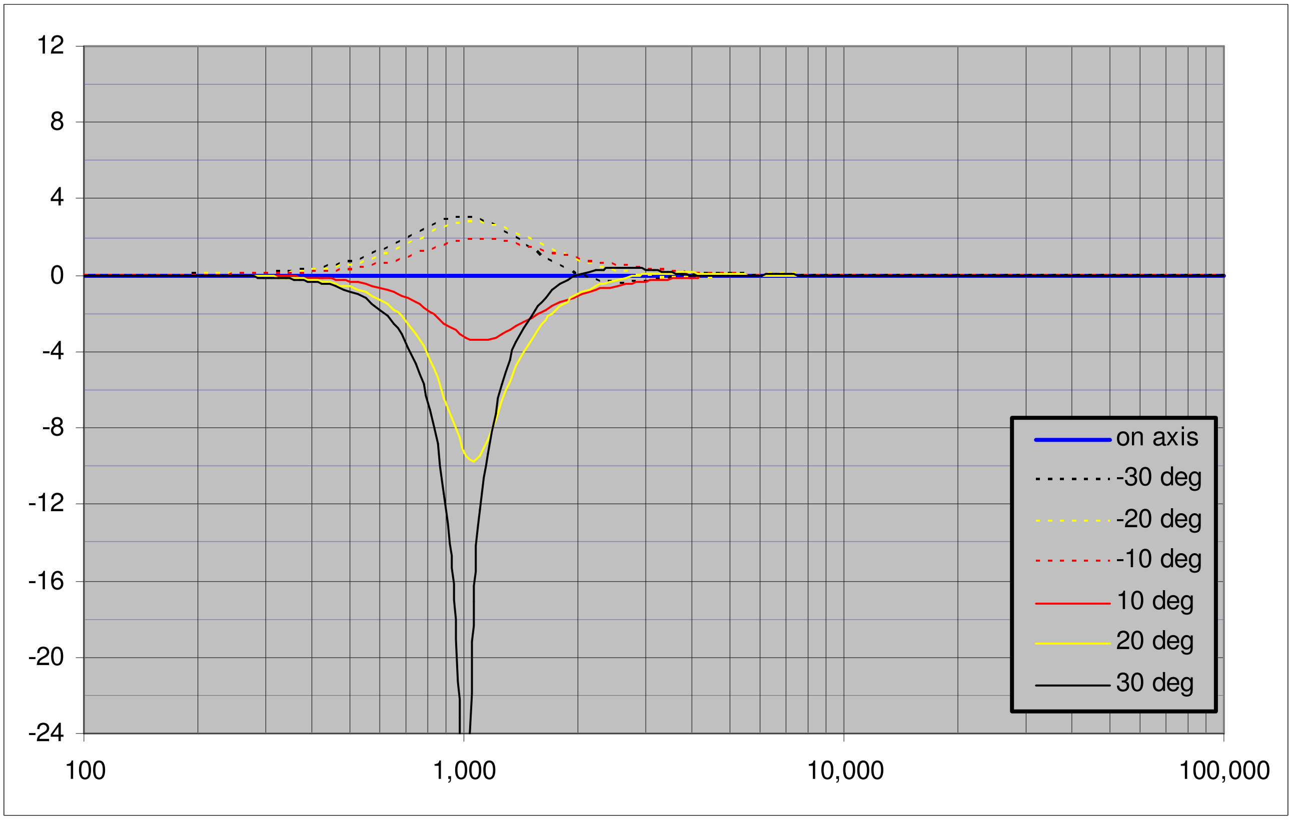
http://www.musicanddesign.com/Power.html

Other types of crossovers display different behavior. For instance both the Butterworth second order (But2) and Butterworth third order (But3) crossovers at 0.5 wavelength driver separation have both peaks and partial nulls in the response. These happen to largely cancel each other out in the power response for the But3 crossover, so that its radiated power is independent of frequency.

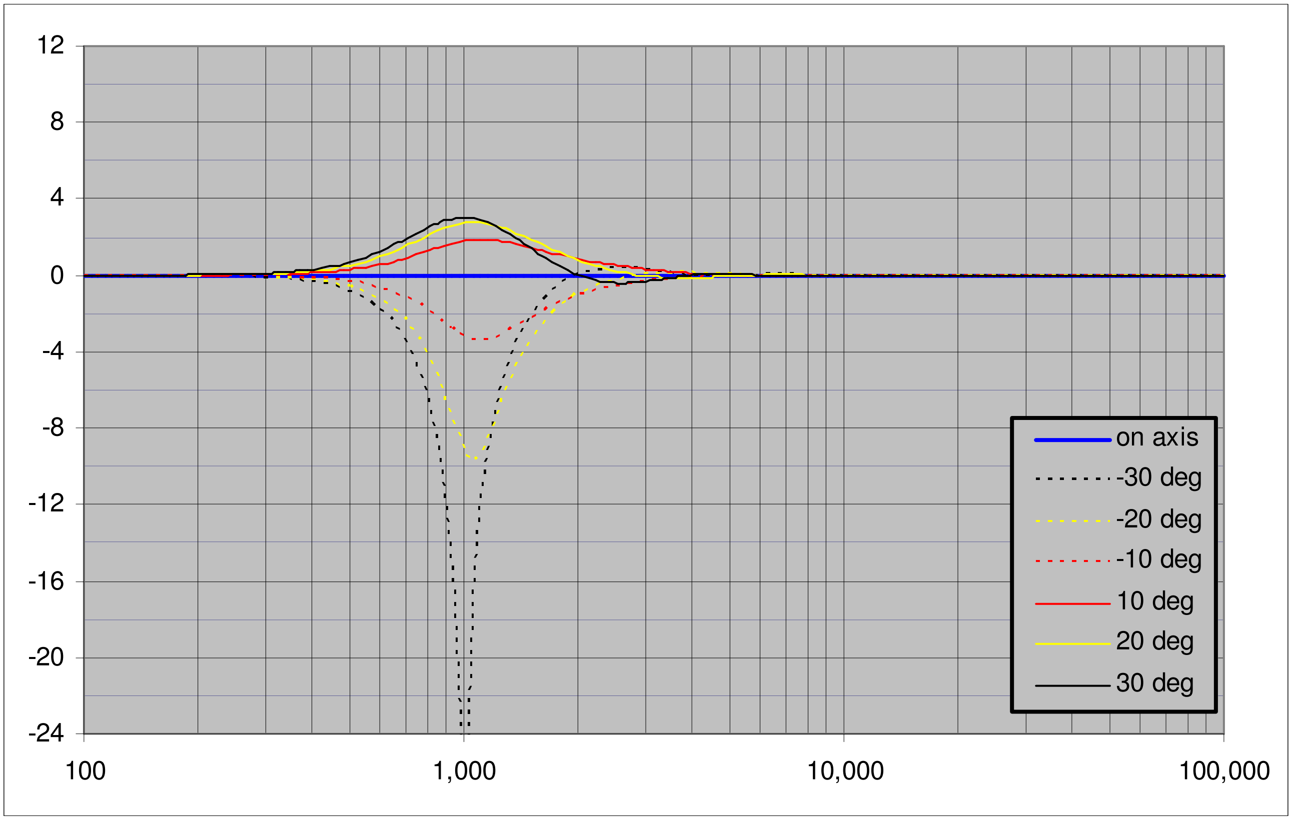
But2 crossover at 0.5 wavelength driver spacing:



But3 crossover at 0.5 wavelength driver spacing, normal tweeter polarity:



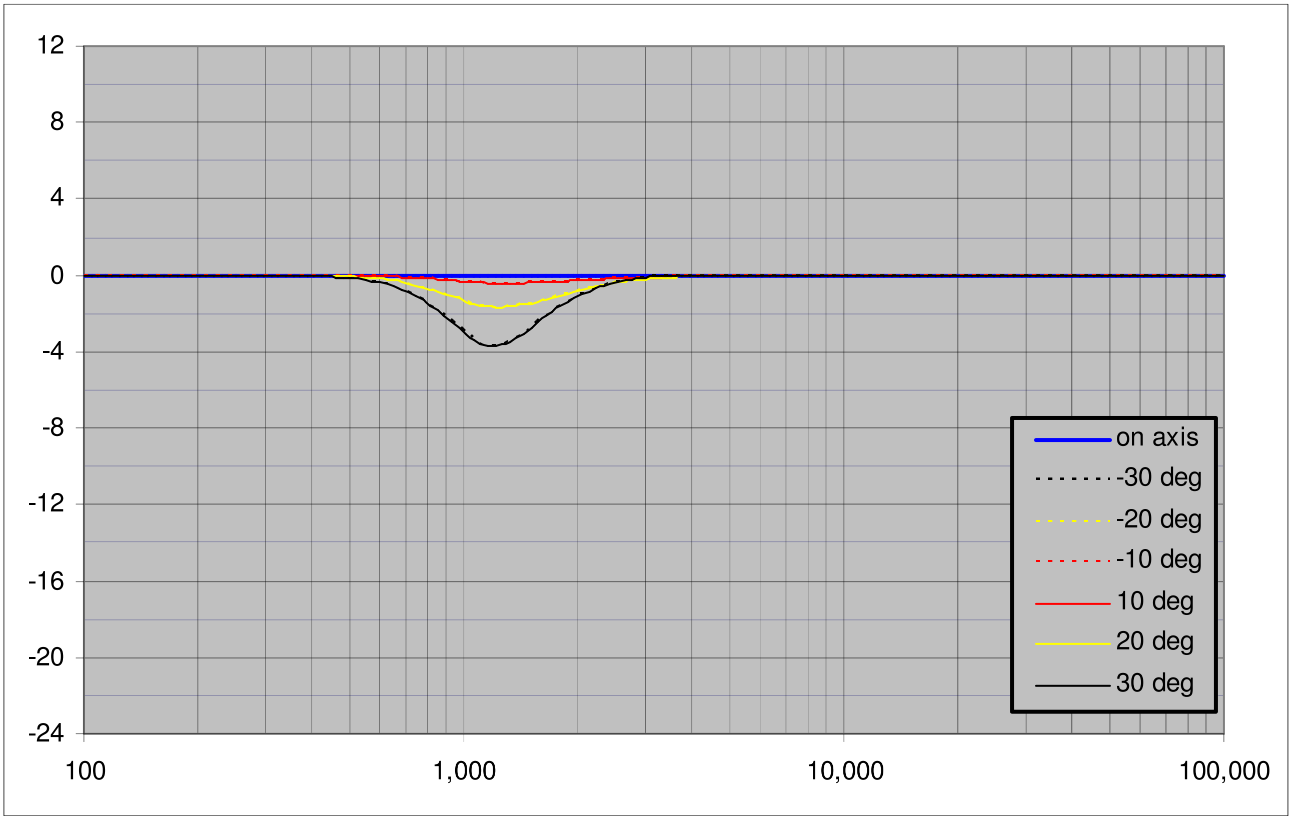
But3 crossover at 0.5 wavelength driver spacing, inverted tweeter polarity:



It is interesting that the on axis response for the But3 crossover type is the same for both normal and inverted tweeter polarity, but the off-axis response behavior is flipped – for the normal polarity, the response partially cancels above the listening axis and peaks below the listening axis, while for the inverted polarity the response peaks above the listening axis and partially cancels below the listening axis.

One additional plot will be presented, of the fourth order Linkwitz-Riley crossover (LR4) with the tweeter connected with normal polarity.

LR4 crossover at 0.5 wavelength driver spacing, normal tweeter polarity:



Again, the pattern is symmetric about the on-axis location in the vertical plane. Note that the crossover region has narrowed with the increase in order, and that the off-axis partial cancelation dips are not as severe.

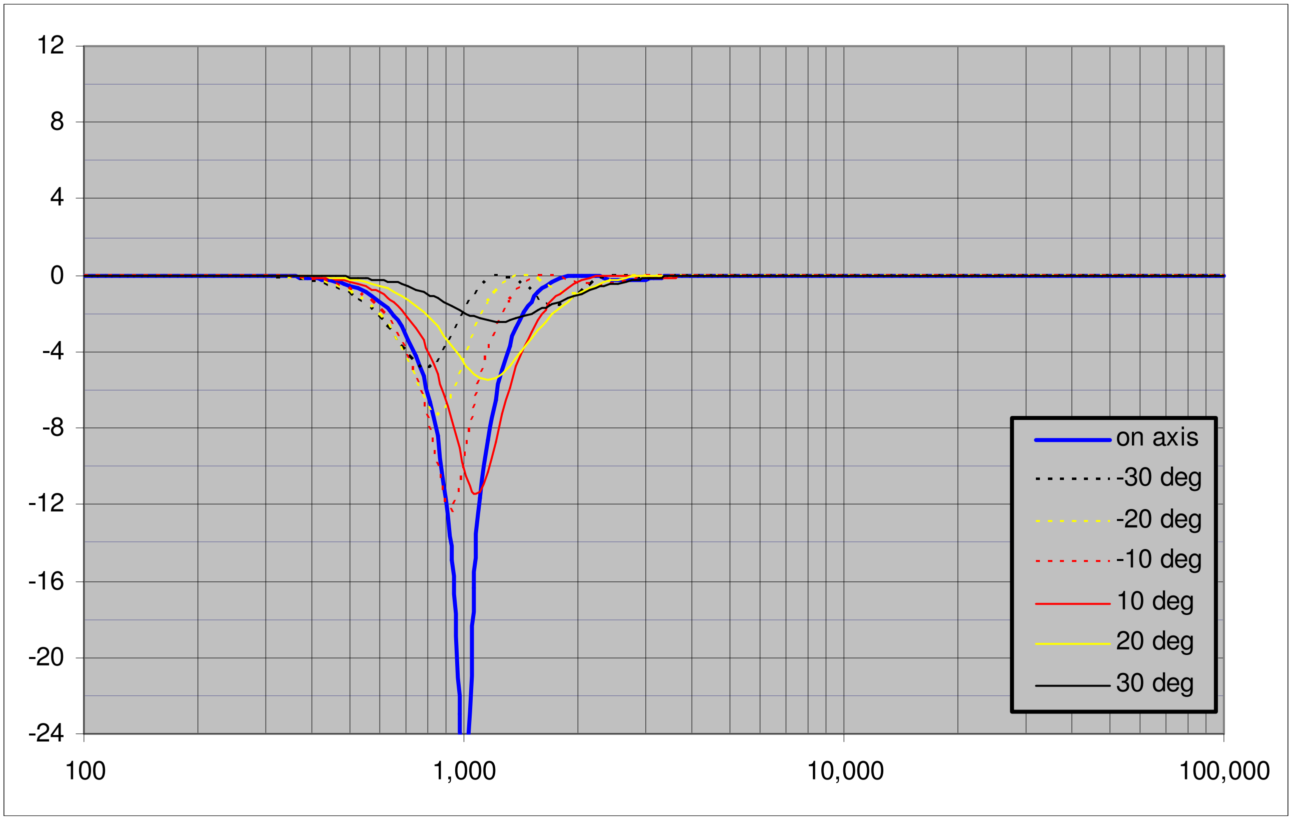
So far, all of the deviations from perfectly flat response on and off axis have come from differences in the distance from each driver to the listening location, which causes phase mismatch resulting in addition or cancellation of the sound waves to some degree.

INFLUENCE OF TIME DELAY (Z-OFFSET)

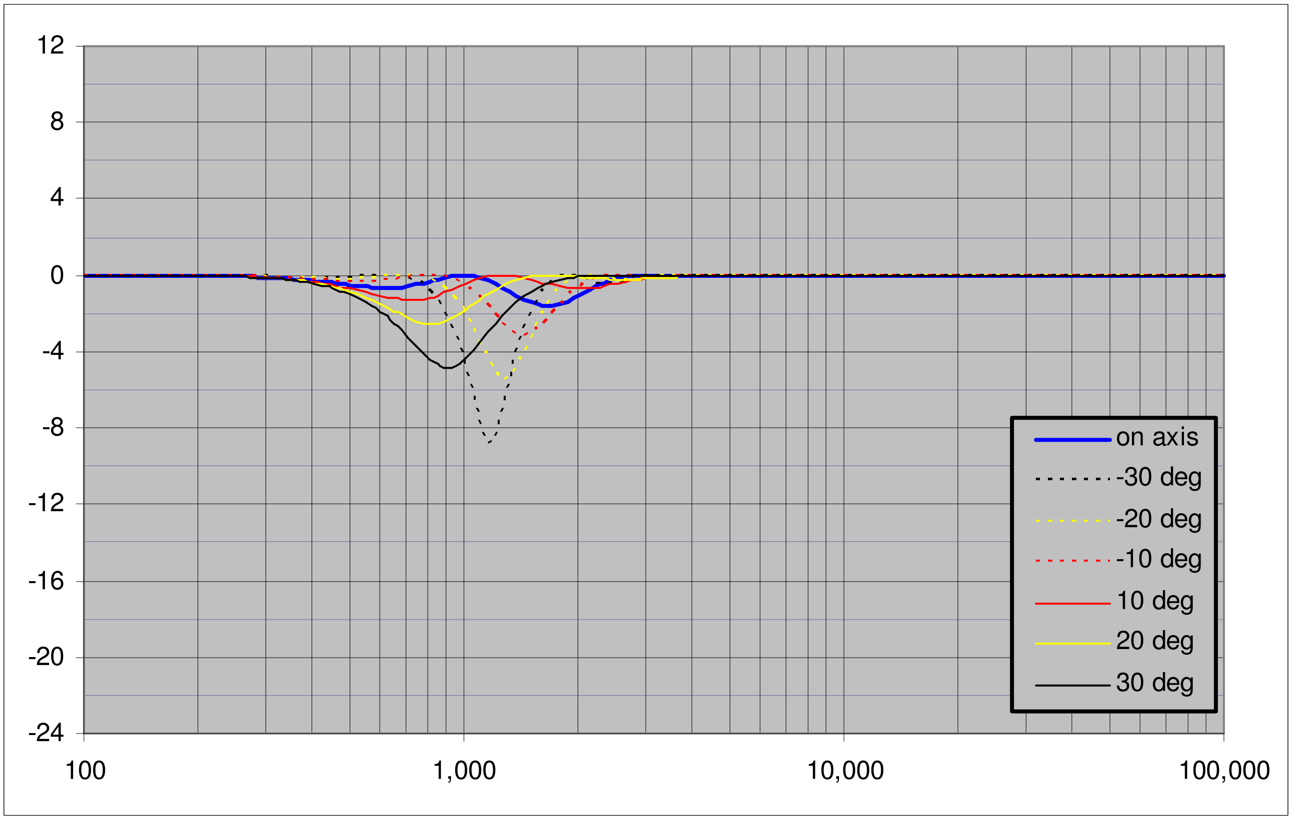
Let's now address how z-offset influences the phase and the response on and off-axis. Recall that z-offset is the difference in the distance between the acoustic center of each driver and the reference plane (the front surface of the baffle). Typically the depth behind the baffle for a tweeter is smaller than that for a mid/woofer, and this causes the midwoofer's signal to be delayed compared to the tweeter. The delay is usually a few hundred micro seconds, however this is enough to cause additional phase disturbances in the crossover region because the mid-to-tweeter crossover is in the low kilo Hertz range and 1k Hz has a period of 1000 µS. Thus, at 1k Hz, the delay is a good portion of the period required for one cycle, and phase can be different by 180 degrees or more.

If one is lucky enough to have "just the right amount"of z-offset, and the crossover region is narrow, the situation can be largely ameliorated just by inverting the tweeter phase, however this depends on many good things happening at the same time! Let's look at an example.
Crossover: 1k Hz, LR4
Vertical driver offset (y-offset): 8 inches (0.204 m)
Horizontal driver offset (z-offset): 6.8 inches (0.172 m)
The z-offset happens to be almost exactly 0.5 wavelength at 1k Hz, thus reversing the phase can mimic a further 180 degree rotation around 1k Hz and the effect of the delay will be removed:

1k Hz LR4 crossover as described in text, normal tweeter polarity:

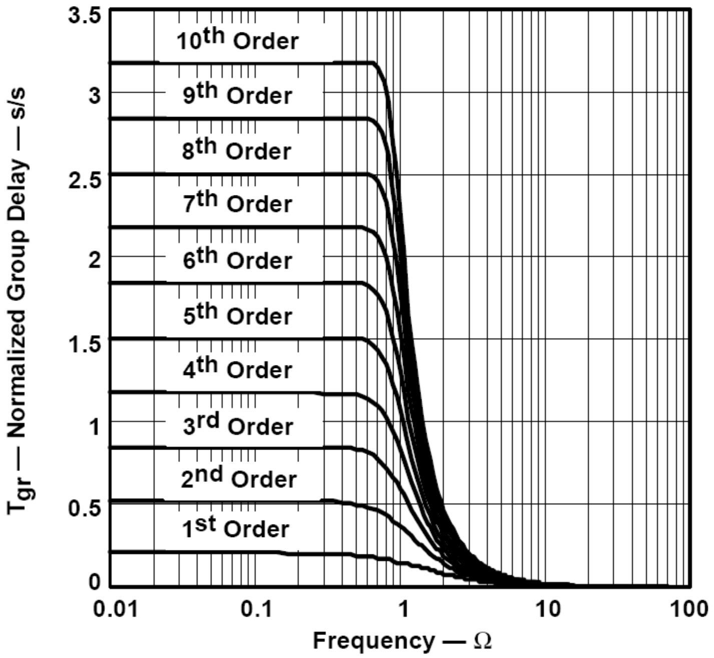


1k Hz LR4 crossover as described in text, inverted tweeter polarity:



As can be seen by comparing the graphs, it happens that the z-offset can essentially be removed because at the crossover frequency the z-offset distance corresponds to approximately one-half wavelength, which generates a 180 degree phase shift. Adding another 180 degree shift by changing the polarity of the tweeter restores the phase match between drivers in the crossover region. A different z-offset would require a different crossover frequency for this magic to work, and each case (each set of drivers) should be considered on an individual basis. Crossover type and order, as well as z-offset, all have some influence and a careful study of each should be done to evaluate how best to configure the loudspeaker crossover.

When some other phase change is required, an active delay circuit can retard the tweeter's signal and restore a phase match with the woofer. Digital delay is best for this purpose, because it delays all frequencies by the same amount. Analog delay via an all-pass network (sometimes referred to as an all-pass filter) is also useful, but requires a bit more planning, because the group delay is not constant for all frequencies. Some designers (Sigfried Linkwitz for instance) have suggested that the corner frequency of the all-pass filter should be equal to the crossover frequency. The all-pass filter corner frequency refers to the frequency by which the delay has fallen to $1/\sqrt{2}$ (or 0.707) of its value at DC (zero Hz). You can think of analog all-pass delay like a low or high-pass filter – both can be formed from cascaded first and second order sections. Each section has a corner frequency and second order stages have a Q, or quality factor. Unlike a high or low-pass filter however, the ideal all-pass filter has completely flat frequency response, but the group delay behavior looks like a low pass filter (group delay, t_{gr}, is related to the derivative of the phase, φ). Higher order all-pass filters allow for higher amounts of delay. Unlike a low-pass filter, however, the all-pass corner frequency and the amount of delay are inversely related, that is for high all-pass corner frequencies, the amount of delay is low, and vice versa. So for high amounts of delay and high corner frequencies, a large number of 2nd order all-pass stages may be required.



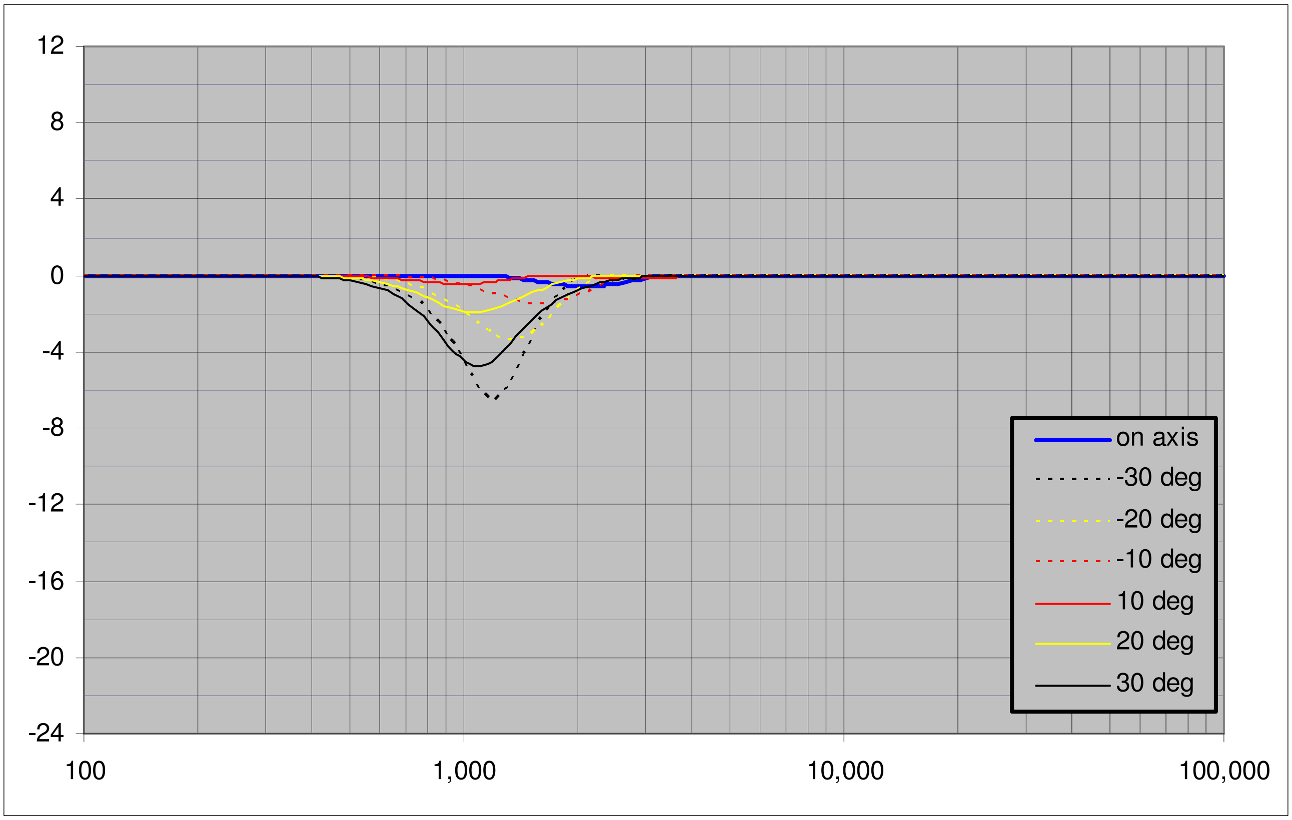
$$t_{gr} = -\frac{d\phi}{d\omega}$$

Analog all-pass filter design chart

Note that the x- axis above is expressed in "reduced frequency" $\Omega = f\,/\,\text{Fc\_ap}$, where Fc_ap is the all-pass corner frequency. To determine what order of filter is required, you multiply the x-axis by Fc_ap and divide the y-axis by Fc_ap, then read off the order that will give you the amount of group delay you desire. For instance, if 500 µS of group delay are needed and Fc_ap is chosen to be 1k Hz, a second order filter is used. Recalling that frequency expressed in Hertz has units of "per second" helps to figure out the units.

Let's look more closely at an example of compensating for z-offset using analog delay.

Consider again our previous example of the LR4 with offsets. If we apply a 1kHz 532 µS all-pass filter we get the following response:



This is very similar to what was observed by inverting the tweeter phase, with a slightly flatter on-axis response. This approach can be carried out for virtually any z-offset to remove the time delay of the mid/woofer output. This is unlike the “magic” tweeter phase inversion, which only works under special circumstances.

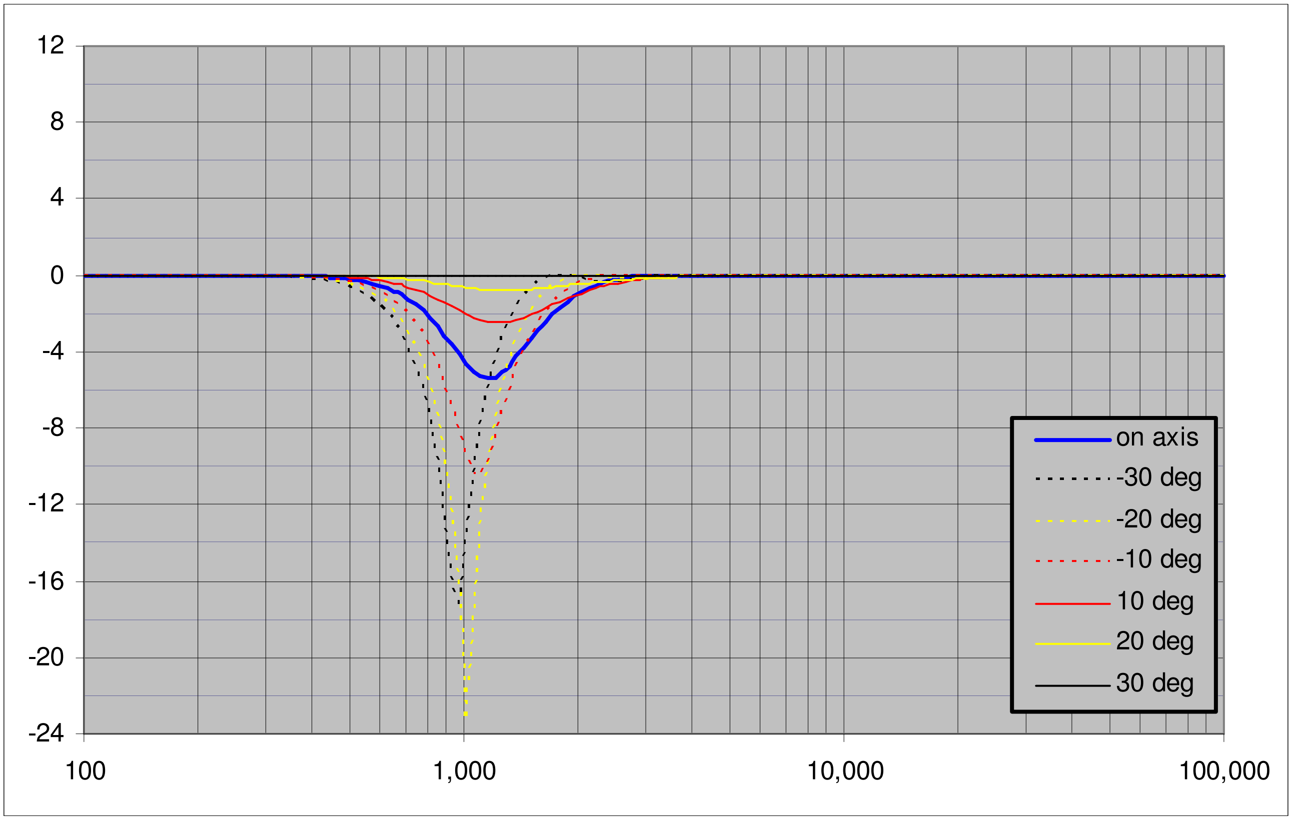
Let’s look at another example of analog delay.

Consider now a loudspeaker with the following:
Crossover: 1.5k Hz, LR4
Vertical driver offset (y-offset): 7 inches (0.178 m)
Horizontal driver offset (z-offset): 4 inches (0.102 m)

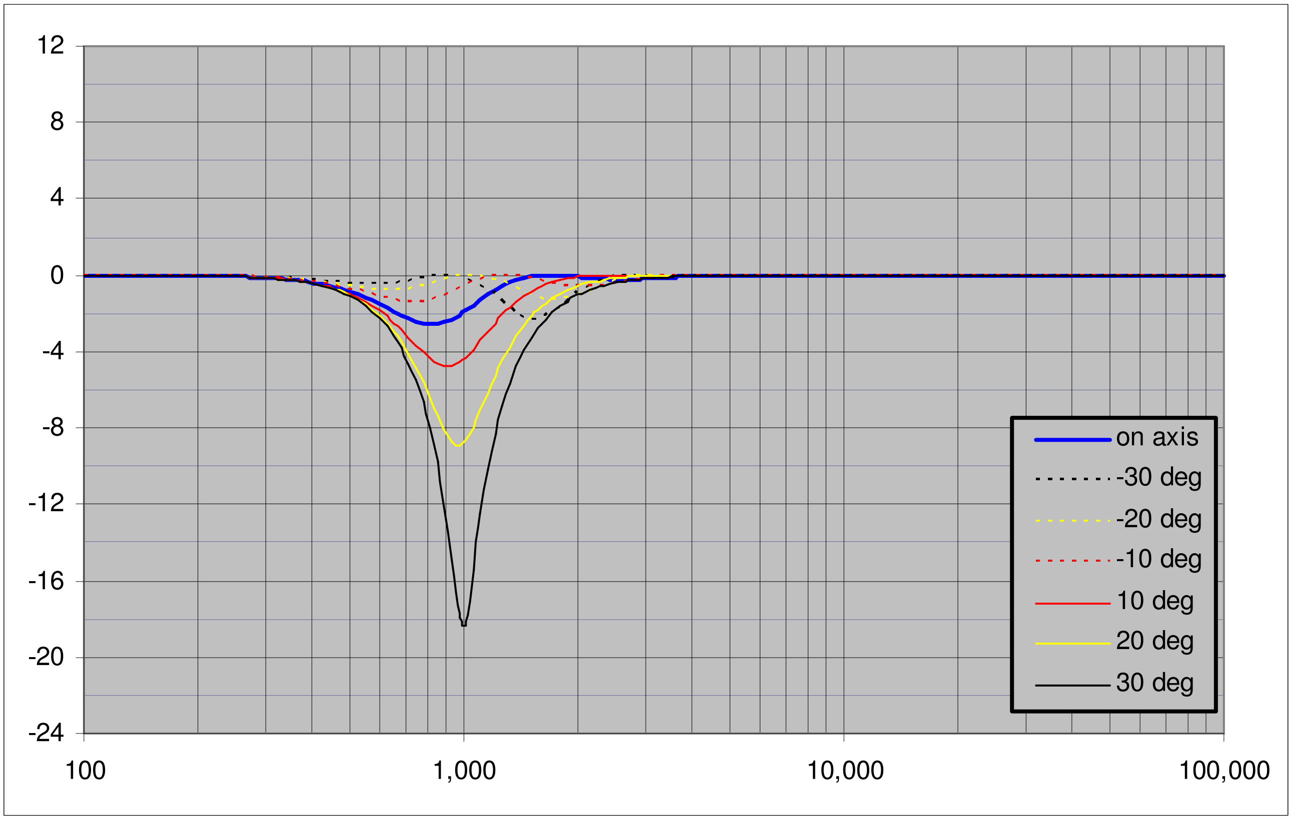
The z-offset results in 297 µS of delay for the mid/woofer. An all-pass filter will be formed in two ways:
(1) by cascading two first order sections
(2) by a single second order stage

Frequency response plots for the loudspeaker without delay, and with the two different analog realizations for the all-pass delay circuit are shown on the next pages. It can be seen that without delay, the on and off-axis response has several partial nulls and the symmetry of the LR crossover response is lost no matter what the polarity of the tweeter. With the delay formed using two first order sections, the on axis response is nearly flat, and the partial nulls off axis are improved, but the LR pattern is still not present. A further improvement is found when the single second order all-pass delay stage is used, and the LR pattern is nearly recovered.

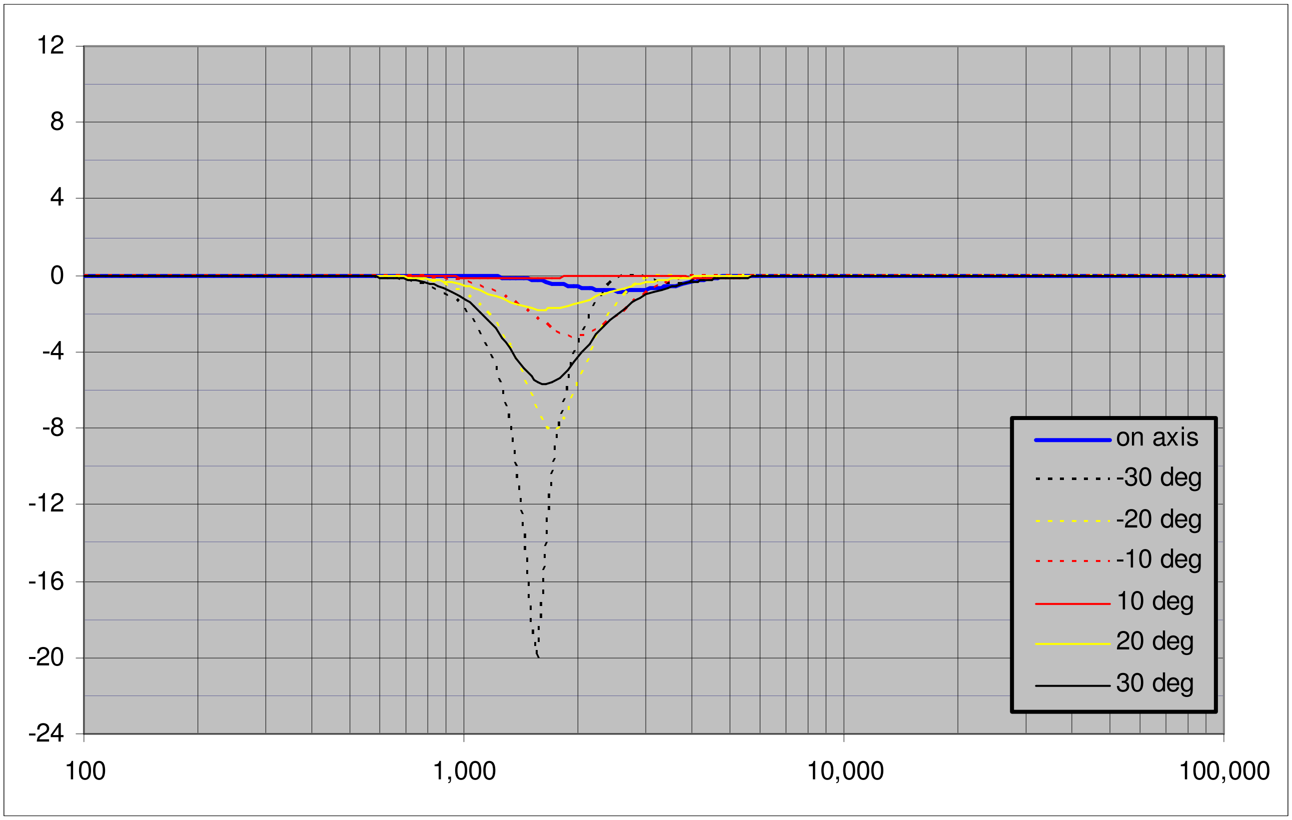
Loudspeaker described in text, no delay, tweeter in normal phase



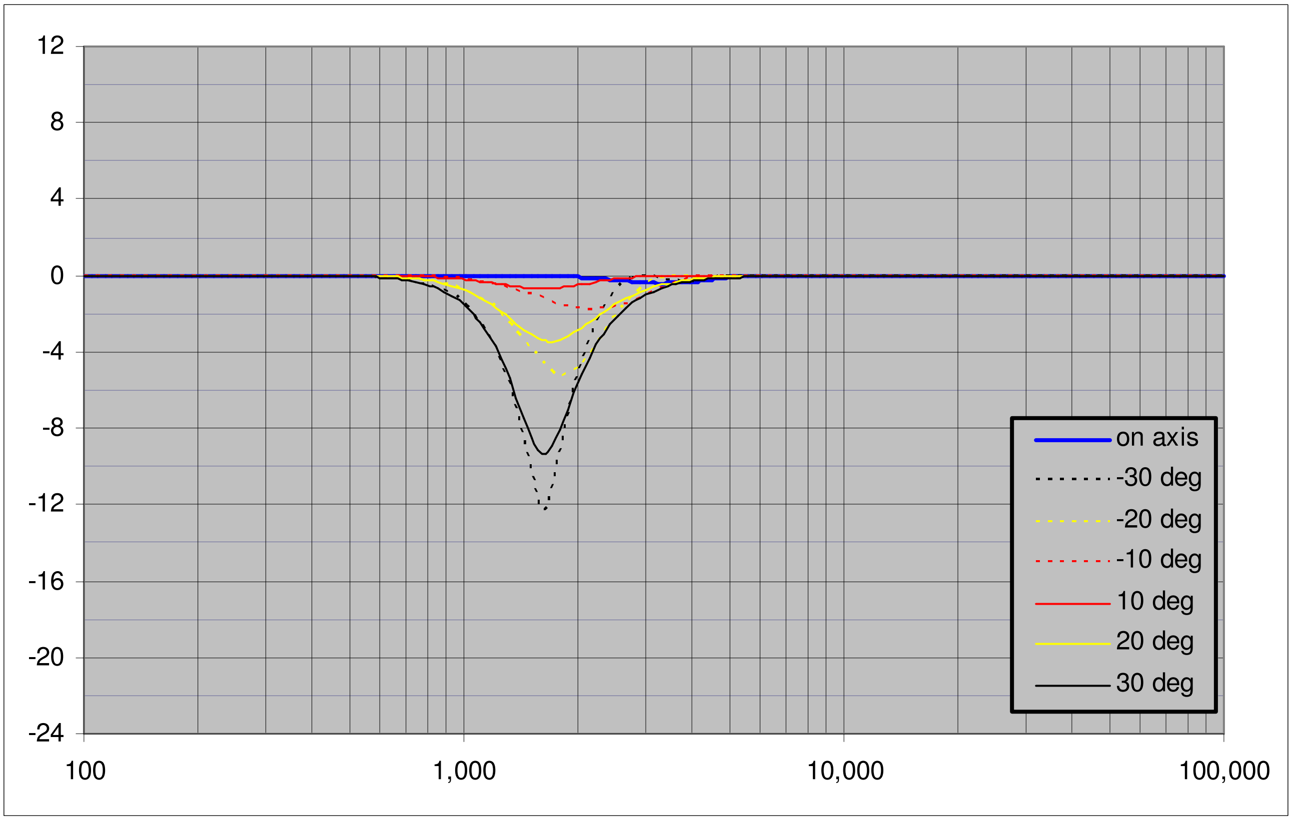
Loudspeaker described in text, no delay, tweeter in reverse phase



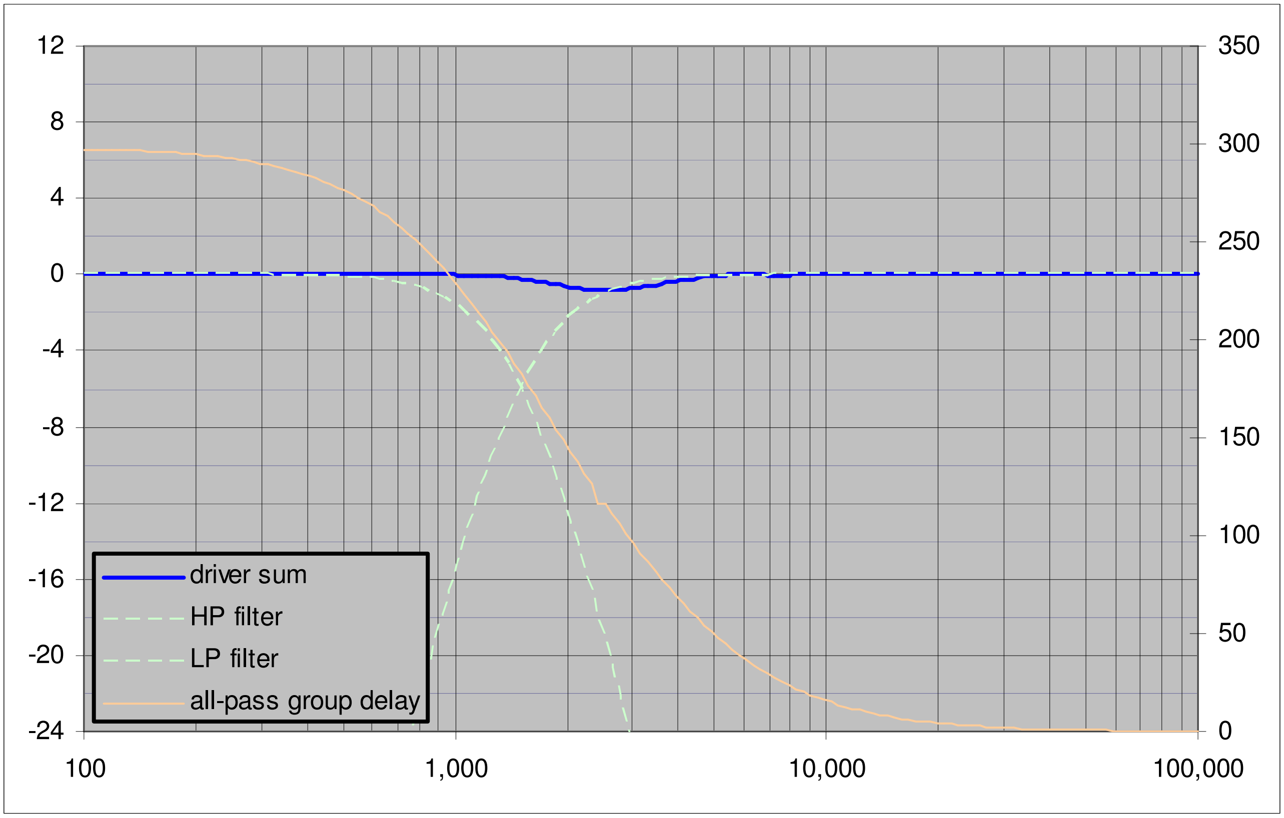
Loudspeaker described in text, 297 µS of delay using two first order stages, normal tweeter polarity



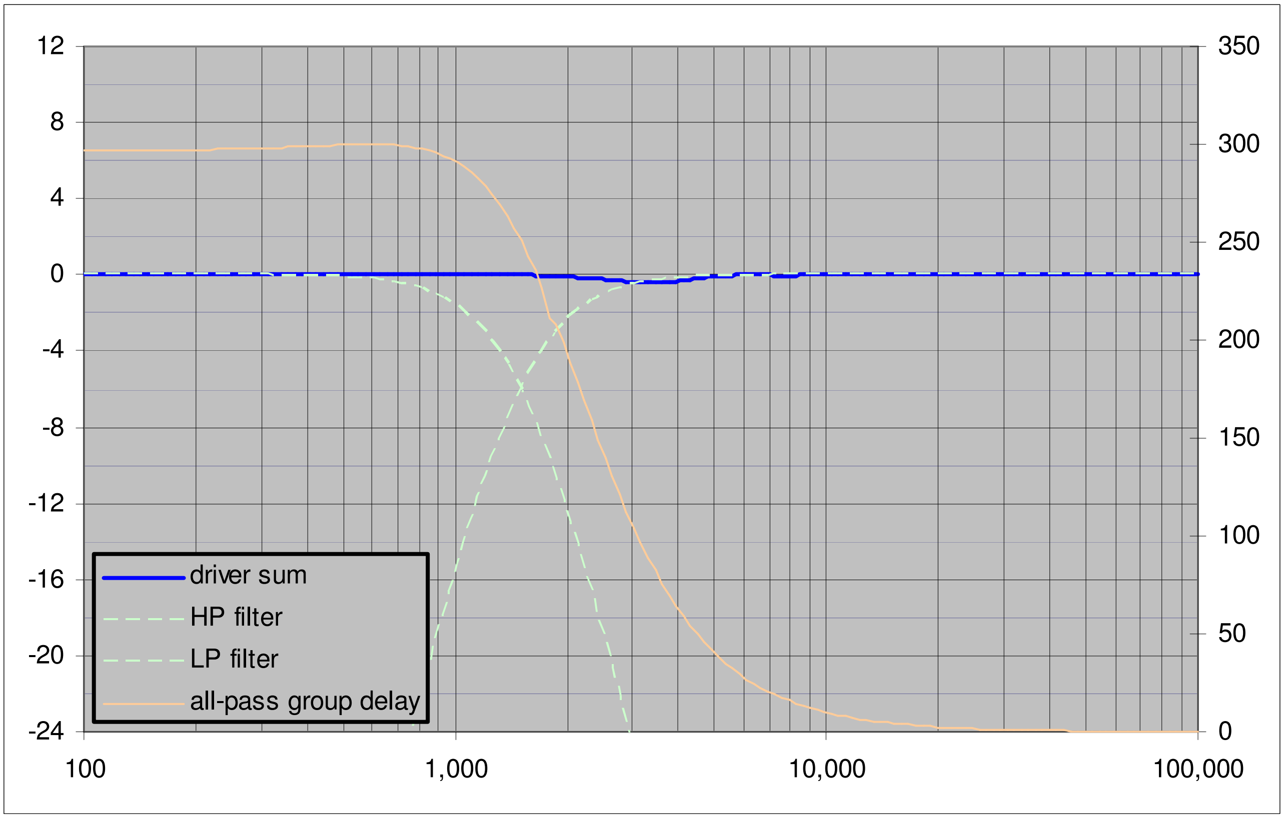
Loudspeaker described in text, 297 µS of delay using one second order stage, normal tweeter polarity



Responses for the delay network formed from two first order all-pass filters



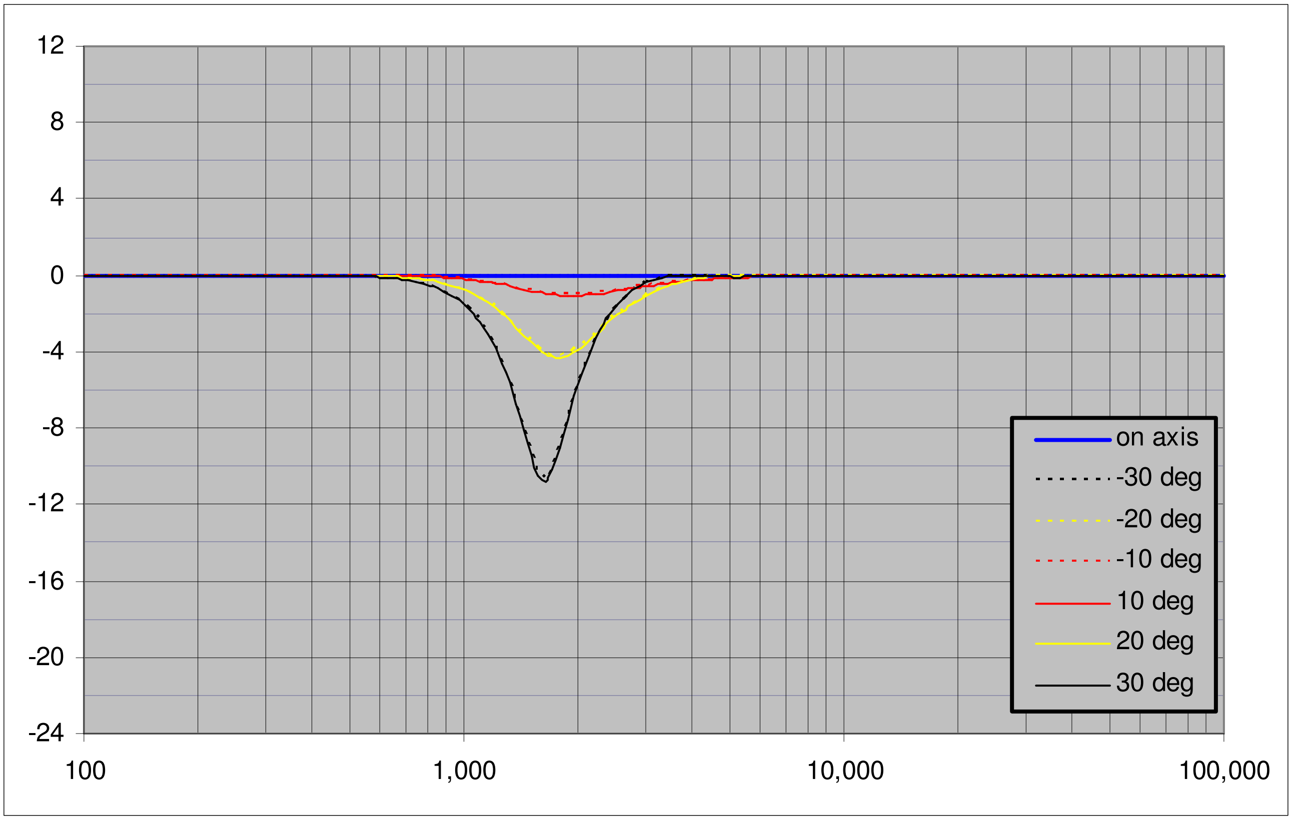
Responses for the delay network formed from a single second order all-pass filter



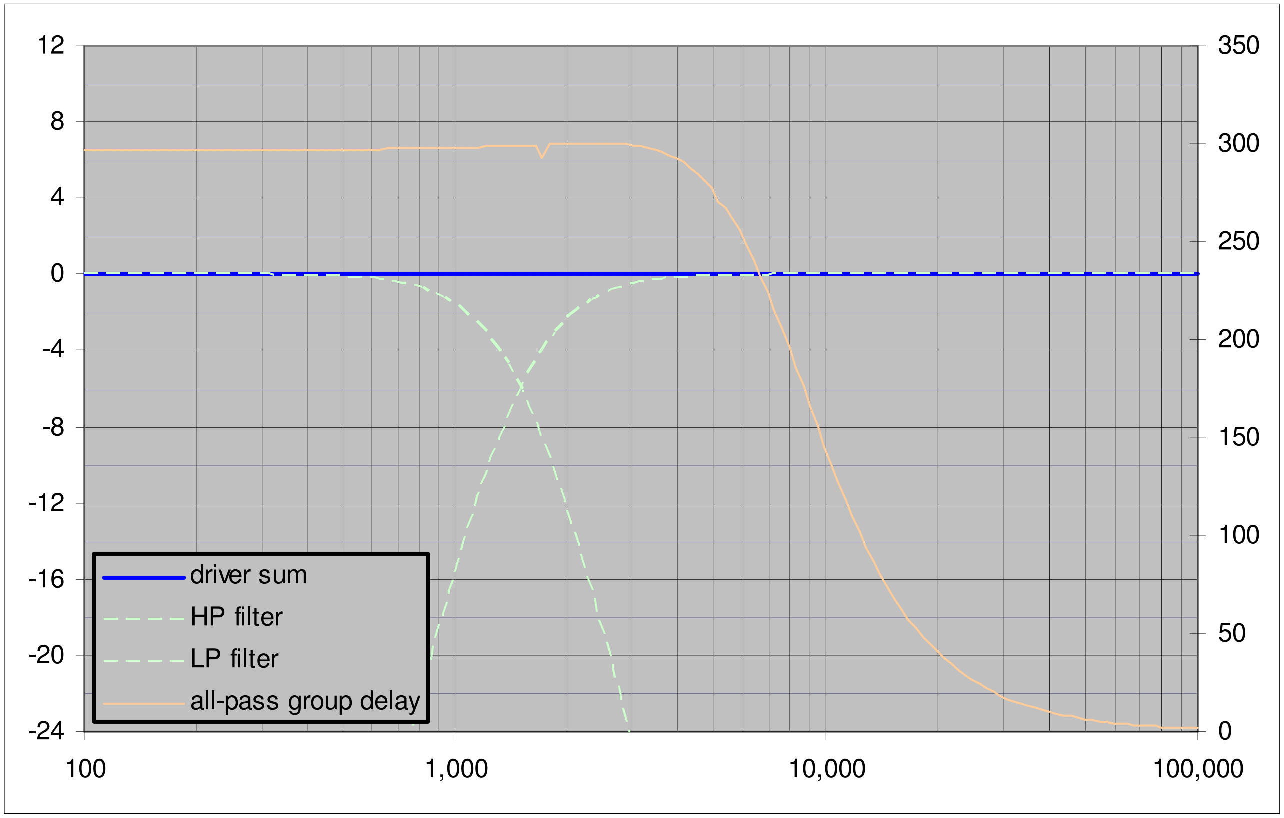
The last two plots, show the group delay for each delay realization. Because the all-pass network's corner frequency is approximately the same as the crossover filter, the group delay is changing throughout the crossover region, especially for the realization using two first order sections. The realization that used the second order section maintains the delay to a higher frequency, and this does a better job of time aligning the output from the two drivers, as shown by the better off axis frequency response pattern.

If we increase the all-pass corner frequency to approximately 7k Hz while maintaining the same amount of delay (297 μS), this moves the transition region of the group delay curve up in frequency past the upper end of the crossover region. This results in even better pattern formation and the LR symmetry is fully recovered (the solid and dashed lines in the plot overlap) however a greater number of second order stages (four) are required.

Loudspeaker described in text, 297 μS of delay using four second order stages, normal tweeter polarity



Responses for the delay network formed from four second order all-pass stages



RECAP

Up to now we have touched on y-offset (driver vertical separation) and learned that it causes the off-axis response to show partial or full cancellation and sometimes response peaks. The extent of these off-axis frequency response variations results from the vertical spacing of the drivers, and moving them closer together decreases these effects. We have also discussed z-offset (time delay) related frequency response anomalies, and showed that these can be reduced or eliminated using an all-pass filter.

The final source of phase change within the crossover region that will be mentioned is the tweeter's own high-pass function. A high-pass filter has phase of 180 degrees for frequencies far below the corner frequency and has a phase of 0 degrees far above the corner frequency. The range of frequencies over which the phase changes from 180 to 0 depends on the Q of the high-pass filter. Recall that a high pass filter is a model for a driver in a sealed box (this is essentially what a tweeter is, too), so in the next section drivers will be modeled as high-pass filters, using some realistic values for Fc and Q, to show the effect of the phase of the drivers on the crossover response on and off axis.

THE EFFECT OF DRIVER PHASE

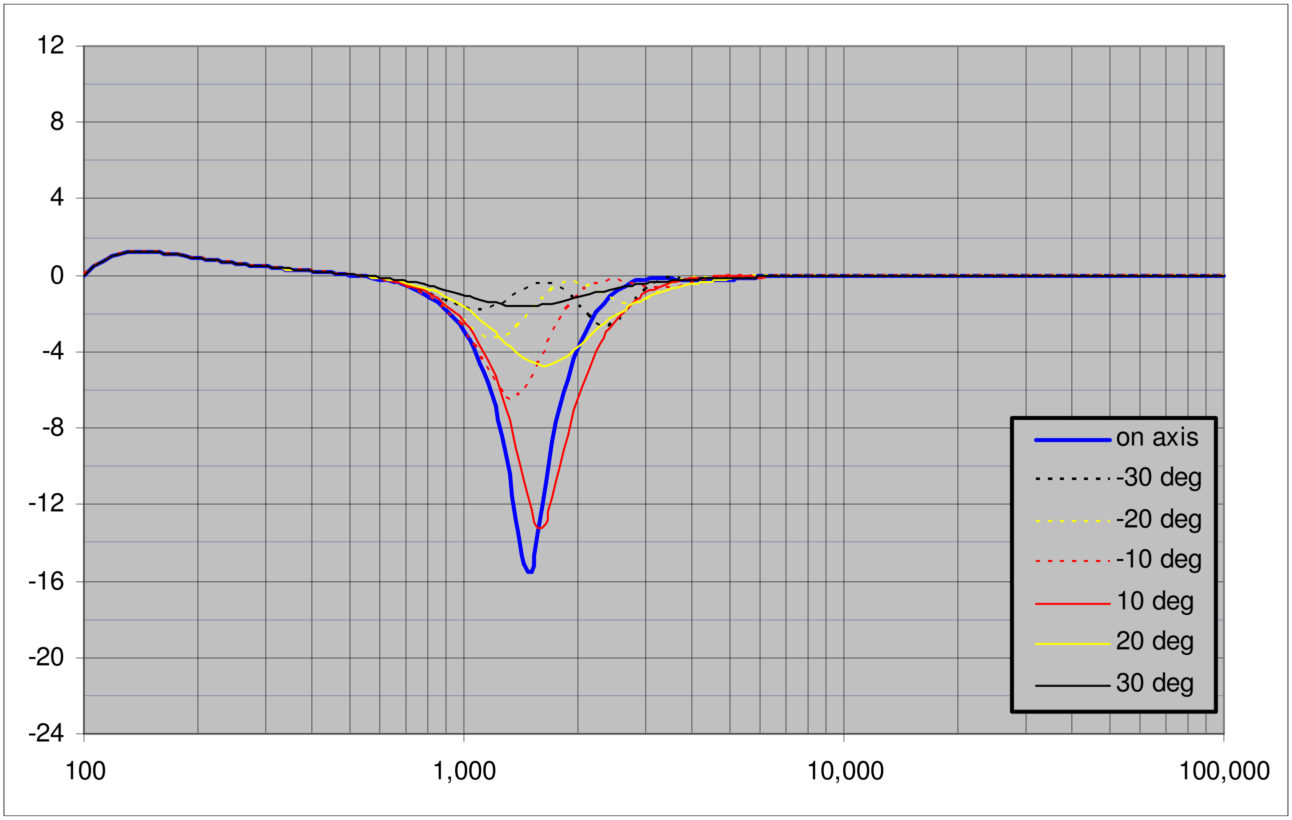
As mentioned in the introduction, we can represent the drivers in the loudspeakers by high-pass second order filters to better represent their phase behavior. Let us take:

Driver #1: Fc = 650 Hz, Q = 0.58

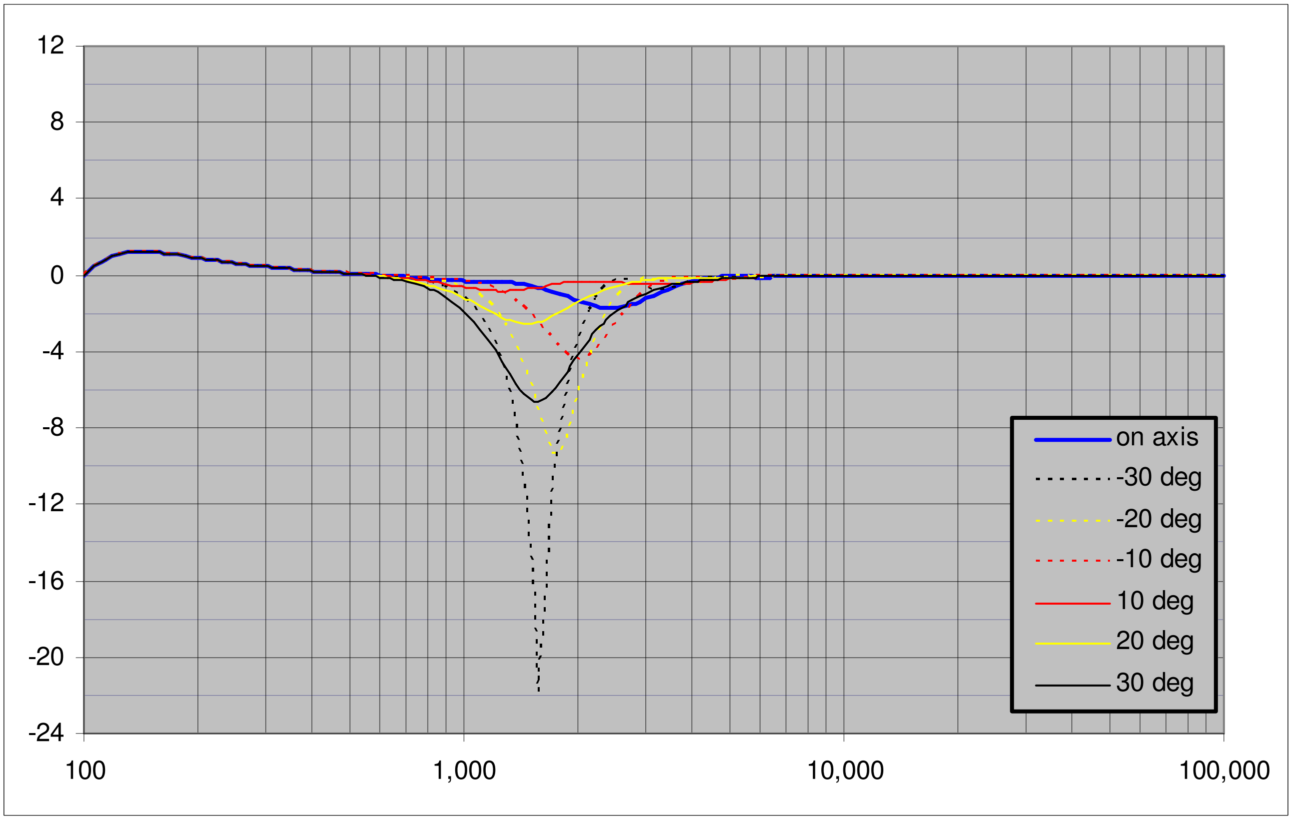
Driver #2: Fc = 100Hz, Q = 1.0

These correspond to a small mid/woofer in a small sealed box (driver #2) and the parameters for driver #1 are those of the Dayton RS-28F tweeter. Let's consider again the LR4 crossover at 1500 Hz with a vertical offset (y-offset) of 7 inches (0.178 m) and a horizontal offset (z-offset) of 4 inches (0.102 m):

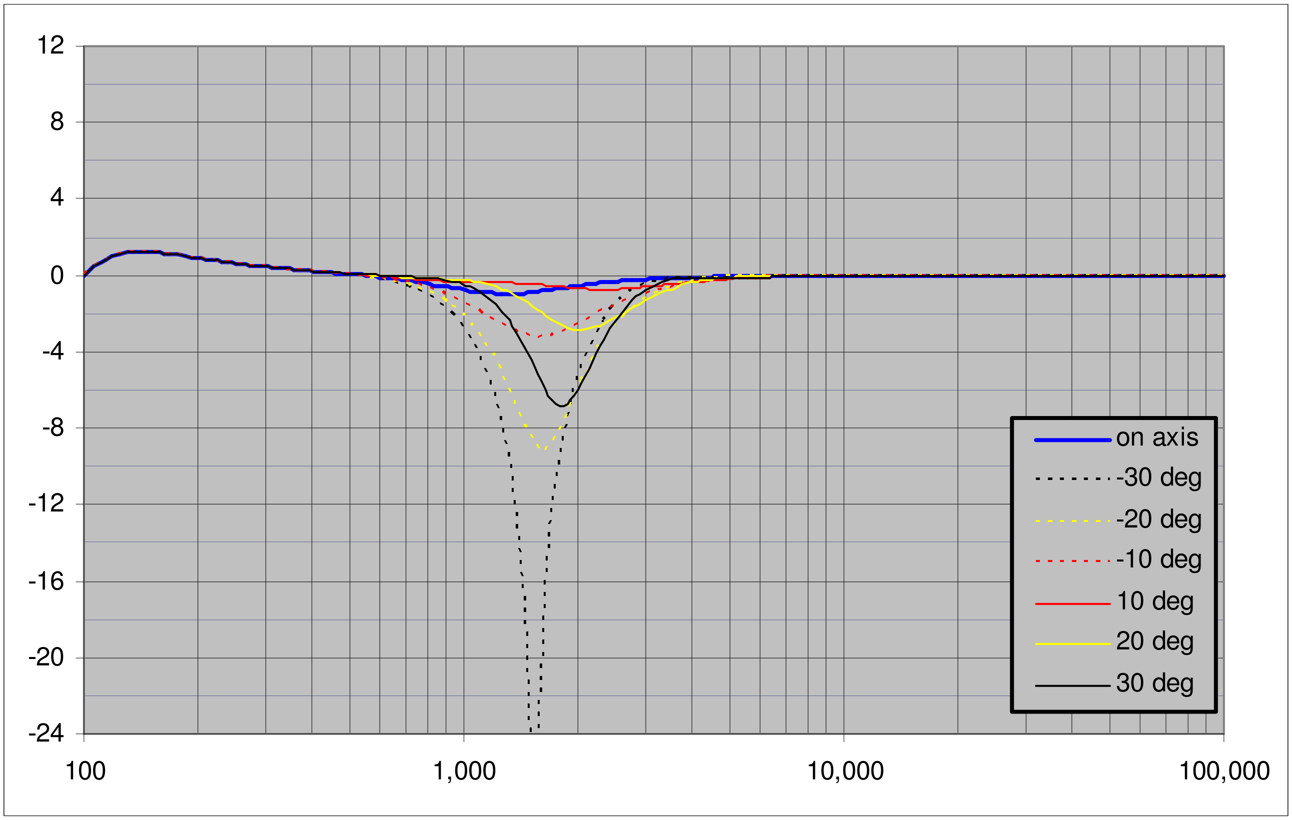
System as described in the text, without phase delay, normal tweeter polarity:



System as described in the text, without phase delay, reverse tweeter polarity:



System as described in the text, 297 µS second order delay at 7k Hz, normal tweeter polarity:



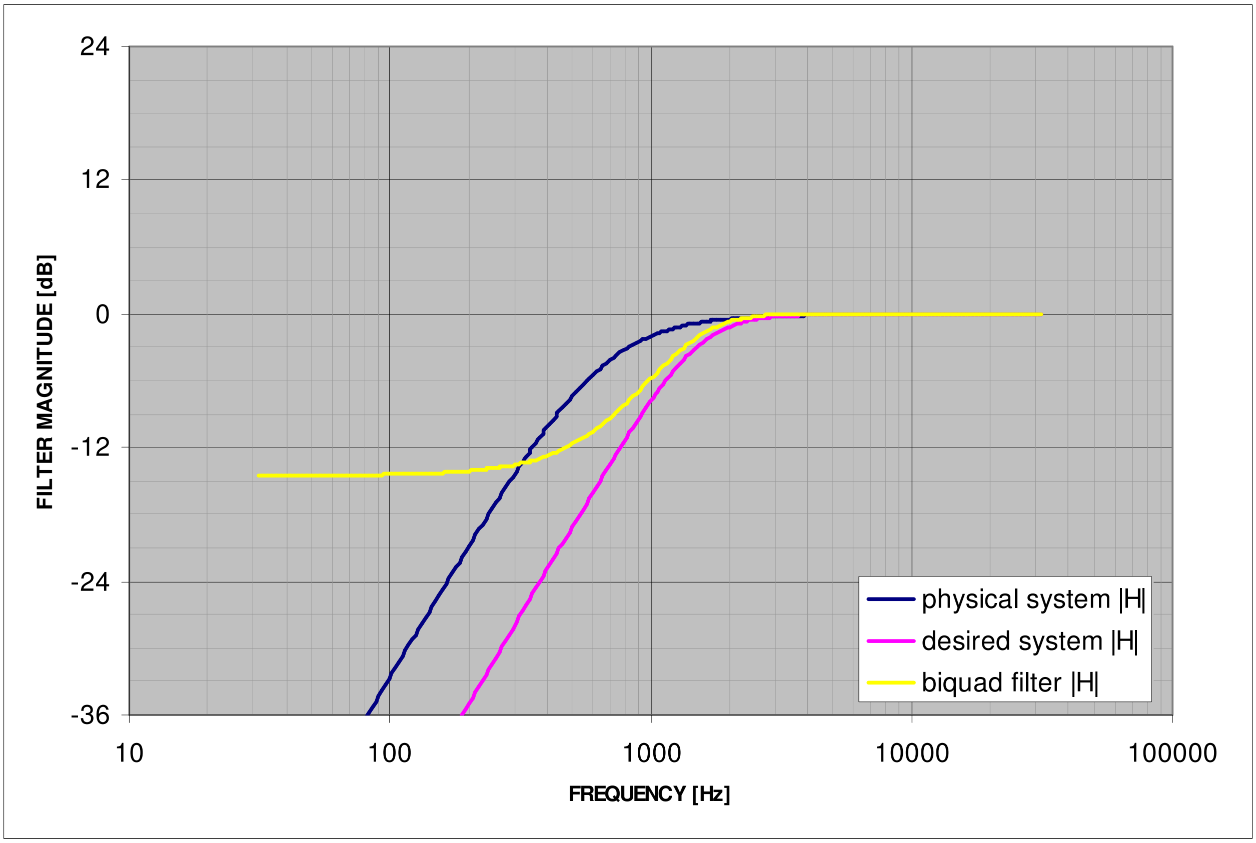
Despite using the second order delay section that previously well compensated for the z-offset, the response has degraded somewhat, and looks similar to when the two first order sections were used. The same problem has occurred – there is additional, unwanted phase rotation occurring in the crossover region. In this case, the source of the rotation is the slow fall of the tweeter's phase to zero because the Q is low (~0.6). Increasing the Q of this driver would decrease this effect, but only slightly. There is also a droop in the crossover region in the on-axis response. This is because the tweeter's low Q frequency response is already about 1dB down at the 1500 Hz crossover point, and this brings down the overall response.

If active electronics are used, there is an easy way to completely remove the problems resulting from the tweeter's phase response. The Linkwitz transform is a filter that implements a bi-quadratic transfer function (BTF). In general, a BTF circuit can "transform" the response of a driver in a sealed box (in terms of its low frequency response, characterized by the parameters F_b and Q) to any new set of F_b' and Q′.

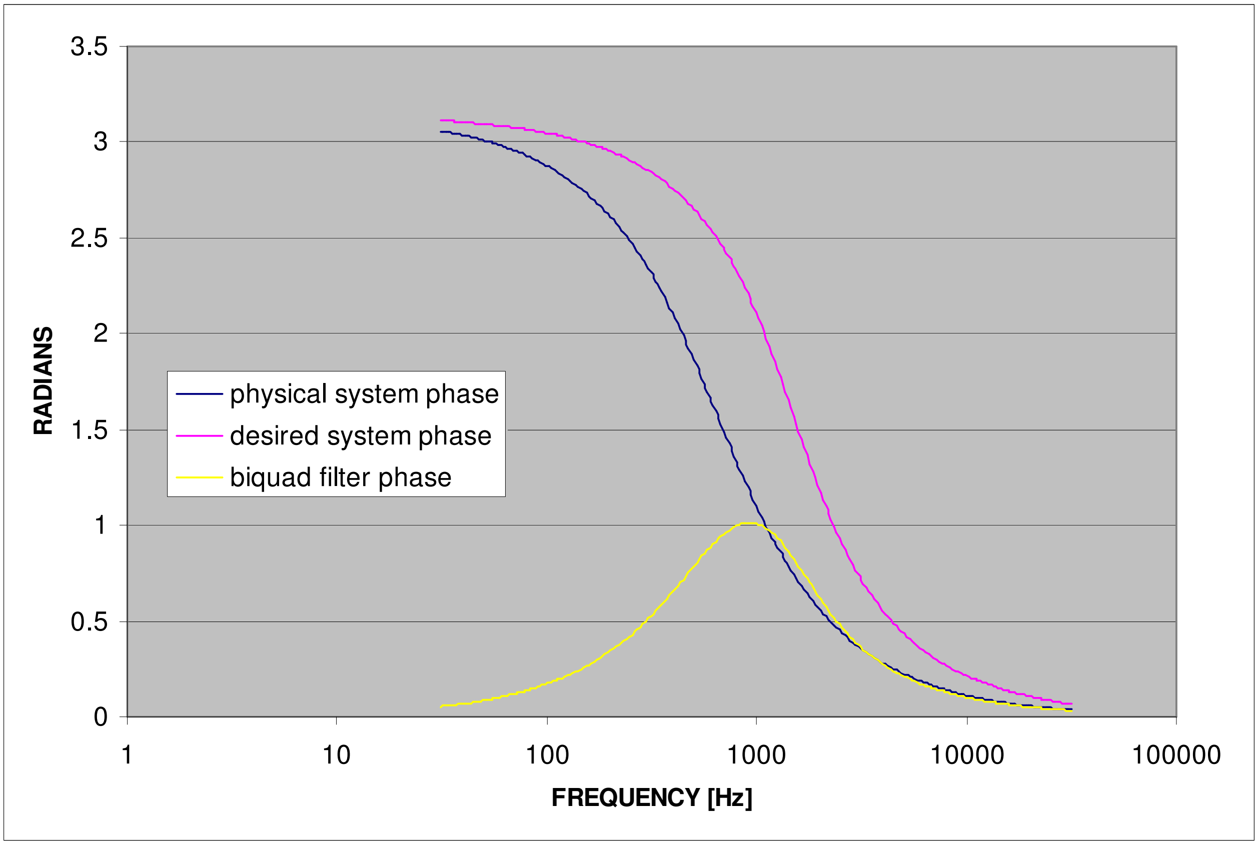
To remove the effect of tweeter phase rotation, one transforms the tweeter's high-pass response in to one that corresponds to one of the second order high-pass stages that makes up the crossover of order 2 or greater. For instance, the Linkwitz-Riley 4th order crossover (LR4) comprises two identical second order staged with Q=0.707 and corner frequency equal to Fc. Rather than using these two stages plus the tweeter, we create the correct acoustic crossover use only one stage and creating the other with the tweeter, transformed with a BTF circuit to have the same Q=0.707 and Fc as the filter stage. So, for the last loudspeaker example, if we use a BTF to change the tweeter's high-pass response to Q=0.707 and Fc=1500 Hz, and combine that with a second order high pass filter Q=0.707 Fc=1500 Hz, the combined acoustic response will be an exact LR4.

The next two graphs illustrate this case:

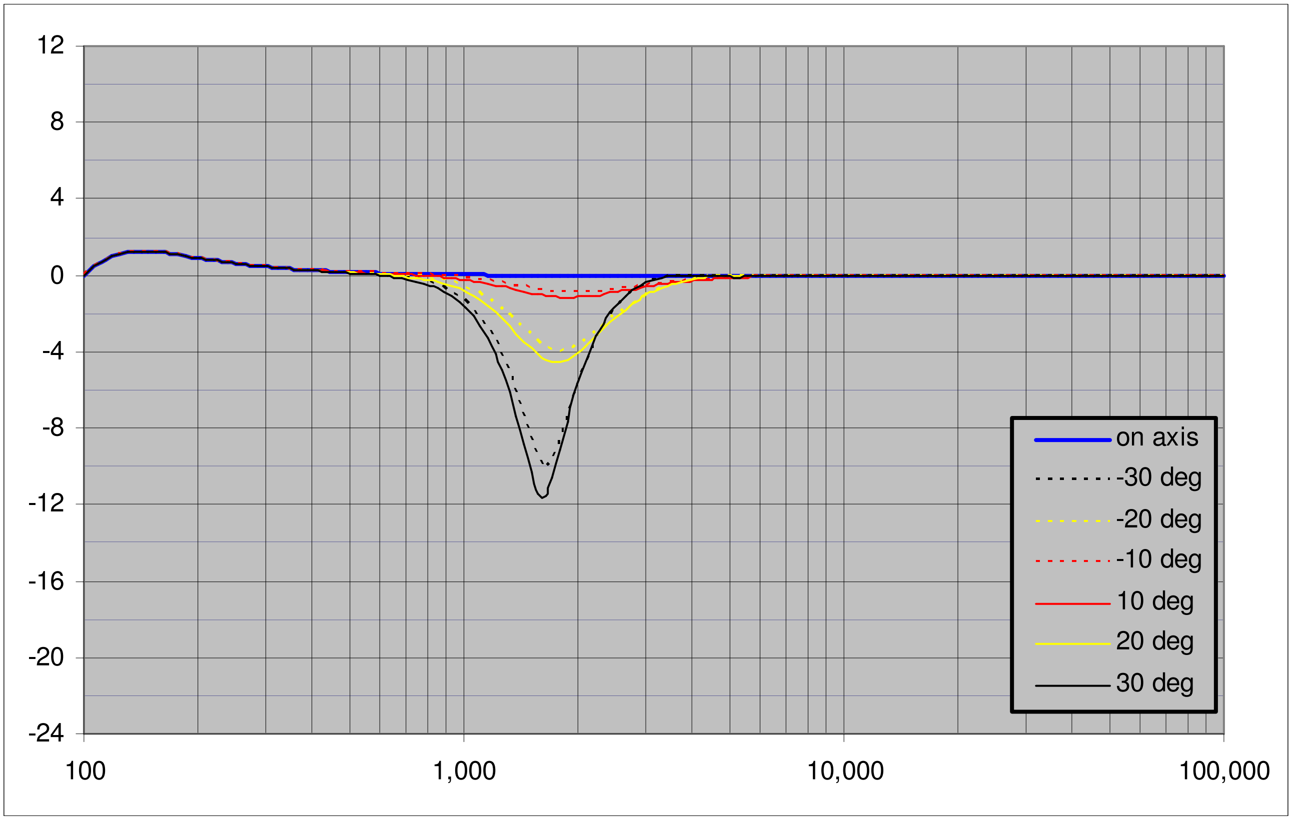
Tweeter response magnitude (|H|) before and after applying the BTF:



Tweeter phase before and after applying the BTF, and BTF phase:



System response with BTF transformed tweeter:



By using the BTF to transform the tweeter and by using it as part of the overall crossover, the effects of the phase disturbance have been removed. In general, this is a very effective and useful technique, especially when the tweeter-to-midwoofer crossover frequency is less than about five times the tweeter's resonance frequency and the tweeter has a low Q.

CONCLUSIONS:

Using various illustrative examples, it has been shown that various factors influence the phase relationship between a mid/woofer and a tweeter within the crossover region. These can cause frequency response irregularities on and off axis.

These effects can be mitigated by:
(1) minimizing the vertical driver spacing

(2) compensating for the time delay caused by a non-zero z-offset between the drivers by:
   a. delaying the tweeter using an analog or digital delay network, or
   b. choosing the crossover frequency so that the z-offset is exactly 0.5 wavelength at that frequency

(3) removing the influence of the tweeter's own phase response by transforming it's low frequency behavior so that it acts as one of the second order crossover stages. This can be accomplished with a biquadratic transfer function filter such as the Linkwitz transform.

By following the above guidelines, a driver with smooth on and off-axis behavior can be created. Because both on and off-axis responses contribute to the soundfield at the listening position, this will ensure that the loudspeaker has a balanced, natural sound.